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THE
RUDIMENTS
OF
NAVIGATION.

RUDIMENTS

OF

NAVIGATION.

THE

RUDIMENTS

OF

NAVIGATION.

Demonſtrated and Illuſtrated in a Plain and Familiar Manner,

By a VARIETY of EXAMPLES.

Together with the

CONSTRUCTION and USE

OF

The Table of Logarithms.
The Lines on the Plain and GUNTER's Scales.
The Table of Natural and Artificial Sines, *&c.*
The Table of Meridional Parts.
The Table of Difference of Latitude, and Departure.

Compiled for the Uſe of the Young Gentlemen on board his Majeſty's Ship MAGNANIME.

By MUNGO MURRAY,
School-maſter of the ſaid Ship.

LONDON:
Printed by D. HENRY and R. CAVE, at *St John's Gate.*
M.DCC.LX.

RPJCB

TO

The RIGHT HONOURABLE

HENRY,

Earl of GAINSBOROUGH,

Viſcount CAMPDEN, *&c. &c.*

MY LORD,

HAVING had the Honour to inſtruct your Lordſhip in the Principles of Navigation, according to the Plan exhibited in theſe Sheets, I humbly conceive there is none to whom I can with ſo much Propriety addreſs them as to yourſelf. Permit me therefore to lay them at your Lordſhip's Feet; and to acknowledge, in this publick

lick Manner, the grateful Senſe I ſhall always retain of your Lordſhip's great Condeſcenſion, and ſingular Favours, conferred upon,

My LORD,

Your LORDSHIP'S *moſt Obliged,*

And moſt Devoted,

Humble Servant,

Depiford,
May, 1760.

MUNGO MURRAY.

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THE RUDIMENTS OF NAVIGATION.

INTRODUCTION.

THE end of the art of Navigation is to inſtruct the mariner how to direct a ſhip's courſe at ſea, and out of ſight of land, to any propoſed port, to which the doctrine of right-angled triangles is alone ſufficient.

We have, therefore, in the following Treatiſe, laid down a few particulars neceſſary to the clear underſtanding of right angled trigonometry, next, ſhewn the affections and properties of ſuch triangles, and then how the ſeveral operations are to be wrought in applying them to navigation.

A Triangle is a figure conſiſting of three ſides, and three angles, the meaſuring of which is the buſineſs of trigonometry; and though it may ſeem moſt natural that this ſhould be performed geometrically, yet, it may, and, in practice, is done with greater accuracy and expedition, arithmetically by the rule of three.

In our firſt chapter, therefore, we have explain'd the principles of geometrical proportion, that being the foundation of the rule of three, which we have divided into four ſections; and becauſe all the operations in that rule are performed by multiplication and diviſion, we have, in the firſt ſection, made ſome remarks on theſe two rules; in the ſecond, the

terms antecedent, conſequent, and ratio, are explained: here we have ſhewn that the equality of ratios is the eſſential property of proportion, and that when four numbers are in a geometrical proportion, the product of the extremes is equal to that of the means: hence, the reaſon of the common operations in the rule of three, by multiplication and diviſion, becomes obvious. But when the numbers conſiſt of more than three or four places of figures, thoſe operations will be tedious; to remedy which, we have, in the third ſection, taught how they may be performed very expeditiouſly by the logarithms, and illuſtrated ſo much of the nature of theſe numbers, as is neceſſary for underſtanding the principles upon which they are calculated: And, in the fourth ſection, we have given the conſtruction of *Gunter*'s lines, with their uſe in the ſolution of queſtions, in multiplication, diviſion, and the rule of three.

The elements of geometry are the ſubject of the ſecond chapter in four ſections: the firſt contains the definitions; the ſecond the theoretical part of geometry; and principally, that altho' all the angles of any plain triangle make 180 degrees, yet the chords, or any other lines which meaſure thoſe degrees, are greater or leſs in different circles, in proportion to the radius of each; and that in ſimilar triangles, the ſides about the equal angles are proportionals: So that if in two ſimilar triangles all the ſides and angles of one of them be known, and either one ſide and the angles, or two ſides and one angle of any other triangle be alſo known, the unknown parts of this other triangle may be found by the rule of three, which comprehends the whole doctrine of trigonometry. The third ſection compriſes the practical part of geometry, particularly, the conſtruction and uſe of the line of chords. The fourth ſection treats of ſines, tangents, and ſecants,

di-

directing how theſe lines on the plain ſcale are conſtructed; whereby it will appear very clearly, that the radius of any circle with the ſine and coſine of any arch of the ſame circle, or the radius with the ſecant and tangent of the ſame arch, always conſtitute a right-angled triangle: ſo that the whole buſineſs of right-angled trigonometry, conſiſts in finding ſines, tangents, or ſecants to any arch, the radius being known; or if the radius and any of theſe be known, thereby to find the quantity of the arch, which we have illuſtrated by examples, adapted to all the different caſes of trigonometry.

The third chapter contains the arithmetical ſolution of all the caſes of right-angled triangles, in two ſections; in the firſt, the principles by which the natural ſines, tangents, and ſecants may be conſtructed are explain'd with their uſe; and the examples that have been performed geometrically in the preceding chapter, are here done arithmetically, both by the natural and artificial ſines, tangents, and ſecants: In the ſecond is ſhewn the manner of ſtating the terms, in all the different caſes of right-angled triangles; and, becauſe all the operations are performed by the rule of three, where there are four quantities concerned, three of which muſt be known, thereby to find the fourth, we have ſhewn, that in the three firſt caſes there is only one, and in the three laſt caſes, two of the unknown terms in the triangle.

To render the doctrine of trigonometry ſubſervient to navigation, the principles of geography muſt be underſtood; which we have therefore explained in the fourth Chapter, in two ſections. The firſt treats of the circles of the ſphere, for which purpoſe, globes are neceſſary to convey a clear idea of the poſition and diſtances of places. But, as in the art of navigation all triangles are delineated on planes, we

muſt find ſome method of deſcribing the circles of the ſphere *in plano* : this is called the projection of the ſphere, which we have exhibited in the ſecond ſection, whereby the latitudes and longitudes of places are determined by plain right-angled triangles.

As all the meridians interſect one another in the poles upon the globe, they will meet in one point, when projected on any plane, as in land-maps; and of conſequence, any ſtrait line will croſs them all, at unequal angles : but as the path a ſhip deſcribes on the ſea, by the direction of the compaſs, makes equal angles with all the meridians, there muſt be ſome expedient found to repreſent them by parallel lines.

We have therefore, in the fifth Chapter, ſhewn how to conſtruct the ſea-charts, in five ſections. In the firſt, the conſtruction and uſe of the plain chart is explained, wherein the meridians being repreſented by parallel lines, the rhumbs, which are expreſſed by ſtrait lines, will croſs the meridians at equal angles, and conſequently, the diſtance, difference of latitude, and departure, always conſtitute a right-angled triangle on this chart; but it is evident, this muſt be very erroneous : for, on this projection, a degree in any parallel of latitude is equal to one on the equator. In the ſecond ſection are demonſtrated the principles of *Mercator*'s chart, and ſhewn, the method of calculating Mr *Wright*'s table of meridional parts, and conſtructing the chart by them. The third ſection contains the conſtruction of all the various caſes of plain and *Mercator*'s ſailing, upon *Mercator*'s chart. The fourth ſhews the manner of calculating the table of difference of latitude and departure, and of working by it a day's work from the log; this being what is aimed at by the whole art of navigation, all the preceding

parts

parts being only preparatory hereunto. And the fifth treats of parallel and middle latitude ſailing.

Tho' the nature and reaſon of the ſeveral particulars taken notice of in this introduction, are fully handled, and clearly demonſtrated, in the following ſheets, yet, as the methods uſed for obtaining the neceſſary *data* are very uncertain, the compaſs itſelf, by which the courſe is directed, being ſubject to variation, it is no wonder if the mariner often falls into very great errors, eſpecially with regard to the longitude; and indeed if the latitude could not be obtained by obſervation, navigation would become very precarious.

In the 6th Chapter we have therefore ſhewn how to find the latitude and variation of the compaſs, by cœleſtial obſervation, all performed by right angled triangles, both geometrically and by calculation, with ſome remarks on the uſual manner of obſerving by *Davis*'s and *Hadley*'s quadrant, and how to allow for the height of the eye above the horizon.

I would not be underſtood, by what I have here publiſhed, to diſapprove of the method uſed by thoſe who teach navigation on ſhore: I only think it will not be ſo convenient for thoſe who are to be inſtructed at ſea, becauſe their duty on deck, in learning to work a ſhip, and clear her for action, beſides the many interruptions they meet with from bad weather, takes up ſo much of their time, that they cannot have leiſure for writing a courſe of navigation; whereas, when they have the rules printed, they need only work all the examples, according to the directions in the book, and write ſuch remarks as ſhall be neceſſary for the perfect underſtanding of ſuch parts as may ſeem to them obſcure; and if the examples in the book are not ſufficient for that purpoſe, as many more may be added, as ſhall enable

able them to give the ſolutions without the book, in the peruſal of which, the following method may be obſerved.

As practical geometry conſiſts chiefly in raiſing and letting fall perpendiculars, after being well acquainted with the method of performing this, they may next learn how to divide the circumference of a circle into degrees, and thereby make and meaſure angles; afterwards they may proceed to conſtruct the line of chords, which they will eaſily diſcover performs the office of the divided circle: and the examples they have in raiſing and letting fall perpendiculars, making and meaſuring angles, are ſuch as will naturally lead them to conſtruct all the caſes of right-angled triangles. They come next to the conſtruction of ſines, tangents, and ſecants; they have here likewiſe examples, wherein the length of the radius, and quantity of the arch, are given, thereby to find the length of the ſine, tangent, and ſecant, in performing which they again conſtruct all the caſes of right-angled triangles: ſo when they come in the next place to what is properly called Trigonometry, they will perceive, it is only giving lines a different name, and the proceſs will in all the caſes be the ſame as before.

The next thing they come to is, the ſolution of all the preceding examples by calculation: in order to which, after they can clearly prove, that the ſines, tangents, and ſecants of the degrees of different circles are in proportion to the radius of each, they are then to deſcribe a circle, whoſe radius may be 100 or 1000 equal parts, and conſtruct ſines, tangents, and ſecants, to every five degrees in the quadrant, each of which is to be meaſured by the ſame ſcale, and their lengths ſet down in a table, regularly ruled for that purpoſe; afterwards they are to work all the caſes in trigonometry, by multiplication

cation and diviſion, and likewiſe by the logarithms; and as the artificial ſines, *&c.* are only the logarithms of the natural, they will eaſily perceive, that there is no occaſion at all for the natural ſines, in computing right-angled triangles.

In the projection of the ſphere I have a globe properly diſſected, which renders the demonſtrations intelligible to ordinary capacities; and when they have thereby gained a clear idea of the method of laying down places, by their latitudes and longitudes, both on the plane of the meridian and of the equator, they will eaſily underſtand the method and neceſſity of enlarging the degrees of the meridian, on *Mercator*'s chart, which they are to conſtruct from the parallel of 50 to that of 61 degrees of latitude, having previouſly calculated the miles in each degree of the meridian, by Mr *Wright*'s method, both by the co-ſines and by the ſecants; and when the chart is conſtructed, they project upon it all the caſes of plain and *Mercator*'s, and the ſubſtance of parallel and traverſe ſailing.

The table of difference of latitude and departure is what they are next to underſtand, and therefore they are to make one page of it, from the artificial and natural ſines: they are then to work all the preceding examples by this table, and are to compare the reſults with the projections, and likewiſe with the operations, by *Gunter*'s lines; after which they are to work the ſeveral days works from the log every day, and from thence transfer them to the journal; and when they are capable of taking the ſun's meridian altitude, they are then inſtructed thereby to find the latitude, and likewiſe the variation of the compaſs, both by amplitudes and azimuths, which they will perceive is done by projecting the ſphere on the plane of the meridian.

As

As this ſmall treatiſe is printed from a manuſcript I compiled for the uſe of ſome young gentlemen of diſtinction in the royal navy, whom I had the honour to inſtruct in the theory of navigation, and, I hope, not without the deſired ſucceſs; I have been forced to depart from the common methods, for the reaſons already advanced, far from imagining, that all the authors who have wrote before me on this ſubject have miſtaken their courſe: for we find, amongſt them, many remarkably ſkilled in all the branches of the mathematics, and from whoſe writings I have collected the ſubſtance of this work. But I flatter myſelf, theſe Rudiments may be of ſome ſervice to thoſe who have not abilities or time to peruſe thoſe copious and prolix treatiſes; and that, when thoroughly underſtood, they may, by their own reading, acquire the whole art of navigation in its utmoſt extent. How far I may have anſwered this purpoſe, I leave to the candid determination of the public.

Explication of the Signs.

$+$ Plus, or more, $-$ Minus, or leſs, $\times$ Multiplied by, $\div$ Divided by, $=$ Equal to, : The ſign of continued proportion, :: The ſign of diſcontinued proportion.

$24+8-10\times6\div4=33$; that is, to 24 add 8, and ſubtract 10 from the ſum; then multiply the remainder by 6, and divide the product by 4; the quotient will be equal to 33.

THE RUDIMENTS OF NAVIGATION.

Of the Principles of ARITHMETIC.

ARITHMETICK and GEOMETRY may be conſidered as the foundation of the ſeveral branches of the Mathematics, it is by them the theoretical part is demonſtrated, the practical part reduced to order, and all the various ſolutions to the moſt intricate caſes obtained.

As we are now going to treat of Navigation, which is, perhaps, one of the moſt uſeful branches of the mathematics, it may be expected that we ſhould explain the principles of the two forementioned ſciences; but as we preſume thoſe who intend to learn navigation, are previouſly acquainted with the fundamental rules of arithmetic, we ſhall paſs over addition and ſubtraction, and only make ſuch remarks on multiplication and diviſion as ſhall be neceſſary to give our readers a diſtinct idea of the doctrine of proportion.

SECTION I.

REMARKS *on* Multiplication *and* Diviſion.

Rem. I.] In multiplication there are three terms, that is, two factors and a product; one of the factors is called the multiplicand, and the other the

multiplier; theſe two being known, the third, or the product, is found by adding the multiplicand to itſelf, or repeating it as many times as there are units in the multiplier.

Rem. II.] In diviſion, there are likewiſe three terms; the diviſor, dividend, and quotient. The firſt two being known, it is the buſineſs of this rule to find how many times, or parts of a time, the diviſor is contained in the dividend; the anſwer to which is the quotient.

Rem. III.] The greater the multiplier is, the greater is the product; and the greater the diviſor, the leſs the quotient: So if the multiplier or diviſor be 1 or an unit, the product will be equal to the multiplicand; and the quotient equal to the dividend; but if the multiplier be a fraction, the product will be leſs than the multiplicand, and if the diviſor be a fraction, the quotient will be greater than the dividend.

Rem. IV.] Multiplication is the reverſe of diviſion, and the contrary; for if any product be made a dividend, and one of the factors a diviſor, the other factor will become the quotient, and if any quotient be multiplied by the diviſor, the dividend will become the product.

It is not our intent here to treat profeſſedly of fractions, we ſhall only obſerve that a fraction is a part of any thing; thus if two loaves were to be equally divided amongſt three people, it is plain this could not be done without cutting each loaf into three equal parts, and then each perſon would have two thirds of a loaf expreſſed thus $\frac{2}{3}$, that below the line is called the denominator, expreſſing the number of parts the thing is divided into; and that above it is called the numerator, ſignifying the number of theſe parts that are expreſſed by the fraction.

Rem. V.] When the multiplier is a fraction, and the

the multiplicand multiplied by the numerator, then if the product be divided by the denominator, the quotient will be the required product: Thus,

32	32	32	32	32	32	32	32
8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	3
256	128	64	32	16	8	24	4)96(24

In every one of which the multiplicands are the ſame; and the multipliers, in the firſt ſix, are each one half of the next before it, and therefore, by the preceding remarks the products will be each the half of the next preceding. Now when the multiplier is 1, the product is 32, therefore when the multiplier is $\frac{1}{2}$ or $\frac{1}{4}$, 32 muſt be divided by 2 or 4, but in the laſt caſe the multiplier is treble the preceding one, and of conſequence the product muſt be treble the preceding product, as by the operation.

As we ſhall have occaſion, in the following Sections, to refer to theſe Remarks, we ſhall here ſubjoin ſome Examples, to illuſtrate what has been ſaid on that head.

Example 1. How many minutes in 29 degrees, ſuppoſing 60 to a degree?

	29
	60
Anſwer	1740

Ex. 2. A ſhip 9 miles diſtant from the Lizard, ſails 5 days on one courſe at the rate of 5 miles in an hour, and then meets another ſhip at ſea, who wants to know her diſtance from the Lizard. It is admitted ſhe had been ſailing 120 hours, which multiplied by 5, makes 600, and with the 9 ſhe was diſtant from the Lizard when the account began, makes 609 miles.

Days	5
Hours	24
Hours	120
Miles	5
Miles	600

Ex-

Ex. 3. A ſhip at ſea in a calm tries the current, which runs at the rate of 2 miles and $\frac{1}{2}$ in an hour, how many miles has ſhe drove in 24 hours? Anſwer 60; for $24 \times \frac{1}{2} = 12$ and $24 \times 2 = 48$ and $48 + 12 = 60$.

Ex. 4. In 678 miles, how many degrees? Anſwer 11° 18′; 60)678(11° 18′

Ex. 5. Suppoſe a ſhip has been drove 36 miles by a current running at the rate of $\frac{1}{2}$ a mile *per* hour, how many hours has ſhe been driving? Here it is plain the diviſor being a fraction, the quotient will be greater than the dividend, ſo the anſwer is 72; $\frac{1}{2})36(72$, for 72 halves make 36.

Ex. 6. Let 10000 be a multiplier, the multiplicand unknown, but the product is 617,0880 required the multiplicand? Here, by remark the 3d, 617,0880 is the multiplicand, for unit neither multiplies nor divides.

Ex. 7. Let 960 be a multiplicand, the multiplier unknown, but we know that if 10000 were multiplied by 617, it would give a product equal to 960 multiplied by the unknown number, which is the multiplier now required, Anſwer $6427\frac{8}{96}$.

for $617 \times 10000 \div 960 = 64270823$.

Rem. 6.] When ſeveral numbers are to be multiplied or divided by one another, it is indifferent in what order the operations are performed, for if 12 be multiplied by 5, that product divided by 4, and this quotient multiplied by 6, and this laſt product divided by 3, we ſhall find this laſt quotient to be 30 by different operations, for 4 and 3 are diviſors, and 5 and 6 multipliers in both operations.

12	4) 12 (3
5	6
4) 60 (15	3) 18 (6
6	5
3) 90 (30	30

SECTION II.

The whole body of mathematics is chiefly concerned in comparing quantities with one another; their mutual relation is called proportion. All quantities may be repreſented either by numbers or by lines, the operations in the firſt are performed by arithmetic, and in the latter by geometry. We ſhall here conſider proportion in reſpect of numbers; in treating of which we ſhall uſe the following ſigns: = equal to, × multiplied by, ÷ divided by, + more or added to, — leſs or ſubſtracted from: So $12 \times 5 \div 4 \times 6 \div 3 = 30$ ſignifies 12 multiplied by 5, divided by 4, multiplied by 6, divided by 3, is equal to 30; and $6 + 8 - 5 = 9$, ſignifies 6 more, 8 leſs, 5 equal to 9. *See the end of the introduction.*

Mathematicians explain geometrical proportion by the terms antecedent, conſequent and ratio. When two things of the ſame kind are compared, the firſt is called the antecedent, the ſecond the conſequent, and when the ſecond is divided by the firſt, the quotient is called the ratio: As ſuppoſing the things compared were miles; 2 the antecedent, and 6 the conſequent, then will 3 be the ratio; but if 6 be the antecedent and 2 the conſequent, then $\frac{1}{3}$ will be the ratio. There can be no compariſon or ratio between things of different kinds, as betwixt miles and hours, for 6 hours cannot be divided by 2 miles.

Inference. If any two of the three terms antecedent, conſequent, and ratio, be known, the third may be found by multiplication or diviſion; if the two firſt be given, as 2 the antecedent, and 6 the conſequent; or 6 the antecedent and 2 the conſequent; the ratio is found by dividing the conſequent by the antecedent; ſo $6 \div 2 = 3$ the ratio; and $2 \div 6 = \frac{1}{3}$ the ratio: If the antecedent and ratio be given, as 2 the antecedent and 3 the ratio $2 \times 3 = 6$ is

is the conſequent, or if 6 be the antecedent and $\frac{1}{3}$ the ratio, then $6 \times \frac{1}{3} = 2$ will be the conſequent, by the 5th remark in the preceding ſection. If the conſequent and ratio be given, and the conſequent divided by the ratio, the quotient will be the antecedent. Let the terms be as before: Then $6 \div 3 = 2$ the antecedent, and $2 \div \frac{1}{3} = 6$ the antecedent. When three numbers are compared, if the ratio of the firſt to the ſecond, be equal to that of the ſecond to the third; as 2 : 6 : 18, they are ſaid to be in a continued geometric proportion, and the terms ſeparated by two points as above, 2 is to 6, as 6 is to 18, for the ratio of 2 to 6 is 3 equal to that of 6 to 18: and 18 : 6 : 2, are likewiſe in a continued proportion, the ratio being $\frac{1}{3}$, ſo 18 is to 6, and 6 is to 2.

When four numbers are compared, and the ratio of the firſt to the ſecond equal to that of the third to the fourth, but unequal to that of the ſecond to the third; as 2 : 6 :: 5 : 15 or 21 : 7 :: 18 : 6, they are in a diſcontinued proportion; the ratio of 2 to 6, or of 5 to 15, is 3, but that is not the ratio of 6 to 5, therefore the ſecond and third terms are ſeparated by four points as above, that is to ſay 2 is to 6, as 5 is to 15; and 21 is to 7, as 18 is to 6; in this laſt the ratio is $\frac{1}{3}$.

It is about four numbers in a diſcontinued proportion that the Rule of Three is chiefly concerned, for if the firſt three be known, this rule directs how to find the fourth, the principles of which may be deduced from the following Theorem.

THEOREM.

When four numbers are proportionals, the product of the extremes is equal to that of the means, and when three numbers are in a continued proportion, the product of the extremes is equal to the ſquare of the middle term, that is the middle term multiplied by itſelf.

Note,

Note, When the numbers are properly placed they are all called terms; the firſt and fourth, are extremes, ſecond and third, the means.

In order to prove the theorem, let a be the firſt, and e the third term, r the ratio; then the terms may be expreſſed thus, a, 1ſt term : $a \times r$, 2d term :: e, 3d term : $e \times r$, 4th term. It is obvious, by rem. 6, ſect. 1, that a, 1ſt term, $\times e \times r$ 4th term $= a \times r$ 2d term $\times e$ 3d term.

Now as any numbers may be applied to theſe letters, this theorem will be univerſally true, for let $a=4$; $r=5$; $e=6$ then the terms will be $4 : 4\times5 :: 6 : 6\times5$, that is $4:20 :: 6: 30$; but $4\times5,\times6 = 4,\times6\times5$, that is $4\times30=120$ and $20\times6=120$.

When three terms are in a geometric proportion, the middle term may be taken twice, as $2:6:18$ may be expreſſed $2:6::6:18$ and then $2\times18=36$; and $6\times6=36$.

Having thus proved the theorem; we come now to the rule of three itſelf; which is as follows.

RULE.

I. Place the numbers in ſuch a manner, that when the fourth is found, the firſt may have the ſame proportion to the ſecond, that the third has to the fourth.

II. Multiply the ſecond by the third term, and divide the product by the firſt term, the quotient will be the number required; that is, the fourth term: The only difficulty will be, as there are but three terms given, how to place them, which we ſhall illuſtrate by the following example.

If a man walks 13 miles in 4 hours, how many will he walk in 24 hours? Here two of the terms are hours, *viz.* 4 and 24, which muſt therefore be the firſt and ſecond terms, becauſe there can be no ratio between things of different kinds, as was before obſerved, ſo ſhall 13 be the third term, which is of

the

the ſame kind with the fourth term, they being both miles. It yet remains to know whether 4 or 24 ſhall be the firſt term; and becauſe we know ſomething about 4, that is he walks 13 miles in that time, therefore 4 muſt be the firſt term; ſo they may be placed thus, 4: 24:: 13. And $24 \times 13 \div 4 = 78$, the fourth term required; as to the operation it is indifferent whether 13 or 24 be the third term, but we ſhall in all the examples in this work, place them ſo that the firſt and ſecond terms may be of the ſame name, ſo ſhall the third and fourth be likewiſe of the ſame name.

The reaſon of this operation appears very plain from the preceding theorem, and the remarks in multiplication and diviſion; for though the fourth term is not known, we are ſure if it were multiplied by 4, the product would be equal to $24 \times 13 = 312$; now we have got a product and one factor, therefore $312 \div 4 = 78$ muſt be the other factor, the fourth term required.

It will be proper to work for anſwers to the following queſtions as we ſhall apply the ſame numbers to triangles.

QUESTION I. If 10000 yards of cloth coſt 6428*l.* what will 960 yards coſt? 10000: 6428:: 960.

II. A privateer coſt 10000*l.* the ſhare of prize money coming to the proprietors was 8391 *l.* one of them had 735*l.* in the common ſtock, what muſt his ſhare of the prize money be?

10000: 8391:: 735.

III. Two partners, A, and B, put 10000*l.* into a joint ſtock, of which A's ſhare was 617 *l.* finding this not ſufficient they added 11917 to the ſtock, what muſt A pay?

IV. If 960 yards of broad cloth coſt 735 *l.* what will 10000 yards coſt? 960: 735:: 10000.

V. If

V. If 960 *l.* laid out in trade in the courſe of 12 years gain 617 *l.* what will 10000 *l.* gain in the ſame time at that rate? 960 : 617 :: 10000 :

VI. If 617 *l.* pay 735 labourers, how much will pay 10000.

735 : 617 :: 10000 :

Theſe examples we preſume ſufficient to illuſtrate the principles of the rule of three; we ſhall therefore proceed, according to the plan, to ſhew how the operations may be ſhortened by the logarithms.

SECTION III.

Of the Conſtruction and Uſe of the Logarithms.

It is not our buſineſs here to calculate tables of thoſe admirable numbers, that being already done to great exactneſs by ſeveral eminent mathematicians. The learned are obliged for this uſeful diſcovery to the indefatigable labour of the noble inventor Lord *Neper*; but it will be proper here to explain ſo much of their nature as may enable us to underſtand the principles upon which they have been calculated.

Logarithms are artificial numbers, adapted to natural numbers, and ſo contrived that the work of multiplication is, by them, performed by addition, and that of diviſion by ſubtraction. From this deſcription the following inferences may eaſily be deduced, *viz.*

Inference I. Every natural number muſt have a proper logarithm, and therefore we muſt have a table to find them by inſpection; it will be ſufficient for our purpoſe to have them to all natural numbers under 10000, which we have in moſt books of navigation.

Inf. II. If the logarithm of any number be increaſed, the correſpondent natural number will likewiſe be increaſed, and the greater the natural number, the greater its logarithm.

Inf. III. If the logarithm of any number be added to itſelf, or, which is the ſame thing, if it be doubled, the ſum will be the logarithm of the ſquare of the natural number, that is, when it is multiplied by itſelf.

Inf. IV. If the logarithms of any two numbers are known; the logarithm of the product or quotient of theſe two numbers may with certainty be found; for the ſum of the two logarithms will be the logarithm or their product, and the difference of the two logarithms will be the logarithm of their quotient.

In order then to make a table of logarithms, we may aſſume any number at pleaſure; for the logarithm of the natural number 10, which let be 1000000, it is certain, by *Inf.* 3, that the logarithm of 100 muſt be 2000000, the logarithm of 1000, 3000000; of 10000, 4000000: Let then 10 be multiplied by 10, its logarithm muſt be doubled, which will give the logarithm of 100. Again, if to the logarithm of 100 be added that of 10, we ſhall have the logarithm of 1000, &c. as in the following table.

Num.	Logarithms.
10	1000000
10	1000000
10 x 10 = 100	2000000
10	1000000
100 x 10 = 1000	3000000
10	1000000
1000 x 10 = 10000	4000000

By this table it is evident that the logarithms of all numbers from 10 to 100 begin with 1; from 100 to 1000 with 2; and from 1000 to 10000 they begin

begin with 3, *&c.* This initial figure is called the characteriſtick or index of the logarithm, and denotes how many figures are in the natural number, which will always contain one figure leſs than there are units in the index: for this reaſon the index is ſeparated from the reſt by a point. The logarithms of all numbers under 10, will be leſs than 1000000, by *Inf.* 2, and contain but ſix figures; but to make them contain the ſame number of figures with the other logarithms, 0 is prefixed to them; ſo 0 is the index of all the logarithms of the numbers under 10. And becauſe 1 or unit neither multiplies nor divides, its logarithm muſt neither increaſe nor diminiſh any other: So the logarithm of 1 is 0000000. In order to find logarithms to all the intermediate numbers betwixt 1 and 10, 10 and 100, *&c.* Thoſe who underſtand the extraction of the ſquare root, may find the logarithm of 9, 7, and 2, by the following method; firſt take half the logarithm of 10, and 2*dly* add the logarithm of 10 thereto; 3dly, take half that ſum, and 4thly, add the logarithm of 10 to each: Thus, firſt extract the ſquare root of 10; 2dly, multiply this root by 10; 3dly, extract the ſquare root of this product; and 4thly, multiply this laſt root by 10.

In this manner we may proceed by adding the logarithm of 10 to that of the root; the half of the logarithm of the ſum will be the logarithm of the root of that product, till the root comes to be above 9. Now the preceding root being leſs than 9, we may add the logarithms of theſe two roots, and half that ſum will be the logarithm of this product, which will be nearer 9 than any of the preceding roots; and ſo, by continual additions and halvings, and extracting the roots of the ſeveral products which the inventor, with great labour, has done to 7 or 8 places of decimals,

decimals, till the root at laſt came to 9; the logarithm of which he found to be 0.954242: After the ſame method we may find the logarithms of 2, 7, 11, 13, 17, 19, and, by adding or ſubtracting theſe, we may find the logarithms of all numbers under 23. The ſame proceſs, may be continued till the table is compleat.

	Num.	Logarithms.
	10	2)1000000
Root of 10	3.163	0500000
10×3.163	31.63	2)1500000
Root of 31.63	5.623	0750000
10×5.623	56.23	2)1750000
Root of 56.23	7.499	0875000
10×7.499	74.99	2)1875000
Root of 74.99	8.66	0937500
10×8.66	86.6	2)1937500
Root of 86.6	9.306	0968750
	8.66	0937500
8.66×9.306	80.59	2)1906250
Root of 80.59	8.977	0953125

In the above operation when the root is 9.306, to the logarithm thereof add that of 8.66, the root next leſs to 9, and halving the ſum we have the logarithm of 8.977, which is nearer to 9; it would be needleſs to continue the proceſs, our deſign being only to ſhew how they may be conſtructed. Now having the logarithm of 9, the half of it 0.477121 is the logarithm of 3: Again, if 0.301030 be the logarithm of two, then the double of it 0.602060 muſt be the logarithm of 4: And becauſe $10 \div 2 = 5$ if the logarithm of 2 be ſubtracted from that of 10, we ſhall have 0.698970 the logarithm of 5; $3 \times 5 = 15$, therefore 1.176091 will be the logarithm of 15, for that is the ſum of the logarithms of 5 and of 3;

and

and for the same reason, 1.397940 is the logarithm of 25, for it is double the logarithm of 5: The logarithm of 7 is 0.845098, to which adding the logarithm of 3, we shall have 1.322219, the logarithm of 21; by the same method we shall find 1.544068 to be the logarithm of 35, and 1.690196 the logarithm of 49.

This we presume sufficient to shew how the table may be made, and that it will answer the end of multiplication and division, and of consequence all operations in the rule of three, which we shall illustrate by the same examples which have been done before by multiplication and division.

Examples in Multiplication and Division.

Example I.

		Logarithms.
Deg.	29	1.462398
Min.	60	1.778151
Min.	1740	3.240549

Example II.

		Logarithms.
Days	5	0.698970
Hours	24	1.380211
Miles	5	0.698970
Miles	600	2.778151

Example III.

		Logarithms.
Mil.	$2\frac{1}{2}$ 2.5	1.397940
Hours	24.	1.380211
Miles	60.0	2.778151

Example IV.

		Logarithms.
Deg.	678	1.831230
Min.	60	1.778151
Deg.	11.3	1.053089

Example V.

		Logarithms.
A ship drives, *dividend*, Miles	36.0	2.556302
Current $\frac{1}{2}$ a mile, *divisor*,	0.5	0.698970
Hours, *quotient*,	72	1.857332

When fractions are to be multiplied or divided, reduce them to decimals, and work as if they were whole numbers, and the index of the result will denote how many figures will be in the product or quotient: As, in Example II, the index of the sum

of

of the logarithms is 2, therefore the natural number corresponding thereto, will have three figures; but because there must be as many decimals in the product as in both the multiplier and multiplicand, one of them must be a decimal, as in the operation in Example V, 1 is the index of the remainder, or difference of the logarithms, therefore there must be two figures in the quotient, and because there must be just as many decimals in a quotient, as the dividend has more than the divisor, which in this case is none, they must be both whole numbers. We presume this method is better than perplexing ourselves with negative indexes.

Examples in the Rule of Three.

The 2d example, about the privateer, when done by multiplication and division would stand thus, 10000 : 8391 : : 735; now because the second must be multiplied by the third, their logarithms must be added, and because the product must be divided by the first term, its logarithm must be subtracted from the sum of the other two logarithms. The terms may be placed in different lines, and their logarithms against them as in the following operation.

As 10000	4.000000
is to 8391	3.923814
so is 735	2.866287
to 616.8	2.790101

Here the sum of the two logarithms is 6.790101, but as the logarithm of the 1st consists only of cyphers, it is only subtracting 4 from the index as above, the nearest logarithm I find in the tables, is 790144, without regarding the index, the natural number corresponding thereto is 6168; but because the index is 2, the last figure must be a decimal After the same manner the other questions may be solved, which we shall omit; but as we have frequent occasion to men-

tion

tion decimals, it will be proper here to ſay ſomething of them.

It was before obſerved, that a fraction is a part of ſomething, and that the denominator expreſſes the number of parts the whole thing is divided into, and the numerator, how many of thoſe parts are contained in the fraction; let a ruler of one foot long be divided into 12 equal parts, then ſix of theſe parts, that is half a foot, will be $\frac{6}{12}$, which is called a vulgar fraction: But if the foot were divided into ten equal parts; the half of a foot would be five of theſe parts: Again $\frac{3}{12}$ or $\frac{1}{4}$ of a foot would be equal to two tenths and half a tenth; which is $\frac{1}{4}$ of a foot divided into ten equal parts; and, in order to expreſs this, every tenth part muſt be divided into ten equal parts, by which the foot will be divided into 100 equal parts, and half a tenth will be $\frac{5}{100}$ and 2 tenths $\frac{20}{100}$, ſo $\frac{3}{12}$ will be equal to $\frac{25}{100}$ and $\frac{9}{12}=\frac{75}{100}$.

When the denominator of a fraction is 1 with any number of cyphers annexed, it is then called a decimal fraction: The denominator is always 1, with as many cyphers as there are figures in the numerator, for which reaſon there is no occaſion to ſet down the denominator, that being known by the numerator, to which there is a point prefixed, to diſtinguiſh it from a whole number. Thus .5.25.666. ſignifies $\frac{5}{10}$, $\frac{25}{100}$, $\frac{666}{1000}$. The advantage of decimals is, that by them multiplication and diviſion is performed as in whole numbers, the only difference is in the value of the product, or quotient, but as it has been noted that there are always as many decimals in the product as in both the factors together, and of conſequence, juſt as many decimals in the quotient, as the dividend contains more than the diviſor, the value of each may eaſily be found. From this deſcription of

vulgar

vulgar and decimal fractions, it is obvious a vulgar may be reduced to a decimal by the following rule; as the denominator of the vulgar is to its numerator, so is 10, or 100, or 1000, *&c.* to the numerator of the decimal; so $\frac{6}{12}$=.5, for 12 : 6 :: 10 : 5, and $\frac{3}{12}$= .25; for 12 : 3 :: 100 : 25, and $\frac{8}{12}$=.666 nearly, 12 : 8 :: 1000 : 666, which may be produced to as many places as you please, and though it will never be exactly equal to the vulgar fraction, yet the difference is so very small that $\frac{8}{12}$ of a foot or 8 inches may be reckoned equal to .666 of a foot.

SECT. IV. *Of* Gunter's *Lines.*

We have in the preceding sections shewn how to work all questions in the rule of three either by multiplication, or division, or by the logarithms; we shall now shew how they may be performed by scale and compasses; in order to which we shall first remark, that addition or subtraction may be performed by a good scale of equal parts, and a pair of compasses, which will be sufficiently illustrated by the two following examples.

Example I. Let the two numbers to be added be 301, and 477.

Draw a strait line AB, and with a pair of compasses, take 301 off the scale; which set off from A to C; take also 477 off the same scale, which set off from C to D; the whole line AD measured on the same scale will be found to be 778, the sum of the two numbers. *Plate* I. *Fig.* I.

Example II. Let it be required to subtract 301 from 1000.

Take 1000 off the scale which lay upon the line from A to B; take also 301 off the same scale, which lay off from B to E, so shall AE, measured on the same scale, be 699, which is the remainder. *Plate* I. *Fig.* I.

Now,

Now, if any two numbers may be thus added or ſubtracted, we may do the ſame by the logarithms of two numbers; the two precedings are the logarithms of 2 and of 3, and of conſequence their ſum is the logarithm of 6. Hence it is plain multiplication and diviſion, and of conſequence all the operations in the rule of three, may be performed by compaſſes, and a good ſcale of equal parts, provided we have a table of logarithms to find the numbers that are to be added or ſubtracted. Mr *Gunter* has conſtructed lines, on which the logarithms may be had without the table, which are upon the ſcales called by his name, *Gunter*'s ſcales; one of them is a line of numbers, marked 1, 2, 3, *&c.* to 9; and the ſame figures on the line, produced, at the ſame diſtance as in the firſt part of the line, ſo there is 1 at the beginning, 1 at the middle, and 1 at the end of the line.

Mr *Gunter* conſtructed this line from the logarithms, and becauſe 0 is the logarithm of 1, the figure 1 is placed at the beginning of the line. Again, as 301030 is the logarithm of 2, it muſt be laid off from 1 to the figure 2; in like manner, the figures 3, 4, *&c.* are placed on the line, by ſetting off the logarithms of theſe numbers from 1 at the beginning of the line; and as 1000000 is the logarithm of 10, 1 is again placed in the middle of the line; and becauſe 1 : 2 :: 10: 20, 1: 3 :: 10 : 30; the figures 2, 3, 4, *&c.* are placed at the ſame diſtance from 1 in the middle, that they are from 1 at the beginning. The lines of equal parts, on the plain ſcale, are only divided into 100 equal parts, which we may call 1000, and take that for the logarithm of 10, and ſo the logarithm of 2 will be 301. We ſhall ſhew, in the next chapter, how to make ſcales of 10000 equal parts, which being conſtructed, we may then take the logarithms by

any of these scales, as suppose by the line A B, *Pl.* I. *Fig.* 1 which is a line of equal parts, and from thence transfer them to the line C D, which will be a line of numbers properly graduated, and is evidently a line of logarithms; in like manner the intermediate divisions may be taken from the logarithms.

The line being thus constructed, our next business must be to read it, or find any number, or rather the logarithm of any number upon the line. Now the first figures may be reckoned 1, 2, 3, that is one, two, three, *&c.* then the same figures on the second part of the line, will be 10, 20, 30, *&c.* if the figures in the first part be accounted 10, 20, 30, *&c.* they will, in the second part, be 100, 200, 300, *&c.* The value of the figures being determined, will also determine the value of the divisions betwixt them, for the spaces betwixt them are divided into ten unequal parts; when figure 1, at the beginning, is one, the intermediate divisions, in the first part, will be tenths of an unit, and, in the second part, they will be units; if the first figure 1 be ten, the intermediates, in the first part, will be units, and in the second part, tens: where the spaces will admit, these ten divisions, which are distinguished by longer strokes, are divided into five or into two unequal divisions, by smaller strokes; the short strokes, when they are four, will be two tenths each; but when only one, it will be only one half of the adjoining long strokes. It will now be easy to find any number upon the line; for, if it were required to find 7, and 14; let the first figure 1 be one; the figure 7 next to it will be seven; figure 1 in the middle will be ten, and counting four of the long strokes, beyond figure 1 in the middle, we have 14. Again, let two numbers be 79 and 129; the first 1 must be accounted ten, then counting nine strokes beyond figure 7, we have 79; now 1 in the middle

middle will be 100, the long ſtrokes betwixt 1 and 2 will be tens, and the ſhort ones two units, each ſo counting two of the long ſtrokes, we have 120, and four ſhort ones make 128; we muſt then eſtimate the half of the diſtance betwixt the laſt ſhort ſtroke and the figure 2, which point will be 129.

The lines being thus conſtructed, we have no more occaſion for the table of logarithms, for we have them upon the line, which therefore may be added or ſubtracted by a pair of compaſſes, and of conſequence all queſtions in the rule of three, may be ſolved by this line; obſerving the following general rule.

RULE.

Place the numbers as before directed, and extend the compaſſes from the firſt to the ſecond term; then placing one foot of the compaſſes, in the third term, the other foot with the ſame extent, will reach to the fourth term.

EXAMPLE.

Let the given terms be 2 : 8 : : 5, the extent from 2 to 8, will reach from 5 to 20, which is the fourth term, here we add the difference betwixt the logarithms of 2 and of 8 to the logarithm of 5, evidently ſame as if the logarithms of 8 and of 5 were added, and the logarithm of 2 ſubtracted from their ſum, which is done when the operation is performed by the pen. As to multiplication and diviſion, if 1 be put for the firſt term, it will be as 1 is to the multiplier, ſo is the multiplicand to the product; and if 1 be put for the ſecond term, it will be as the diviſor is to 1, ſo is the dividend to the quotient, which is in effect the rule of three; ſo if it were required to multiply 8 by 5, it would be 1 : 8 : : 5 : 40, and the extent from 1 to 8 would reach from 5 to 40. Again, if it were required to divide 40 by 2, it would be 2 : 1 : : 40 : 20, and the extent from 2 to 1 will reach from 40 to 20. We

We might add more examples, but to make us thoroughly acquainted with this line, it will be proper to work ſeveral examples with the pen, and prove them by this line; and when the product does not exceed three figures, the reſults will agree.

CHAP. II.

Of GEOMETRY.

WE have in the preceding chapter explained the doctrine of geometric proportion by numbers; we come now to ſhew how this may be done by means of lines, without any arithmetical calculation.

SECTION I. *Geometrical Definitions.*

1. A definition is the perfect explanation of any word or thing ſuppoſed not to be underſtood.

2. A circle is a figure or ſpace contained within one curve line, and in practice is deſcribed by a pair of compaſſes.

3. The center is the point where one foot is fixed, the other being carried about deſcribes the curve.

4. The circumference or periphery is the curve line deſcribed by the moving point, as A B D E. *Pl* I. *Fig.* 2.

5. The radius or ſemi-diameter, is any ſtrait line drawn from the center to the circumference, as C A, C B, C D, C E, of which there may be any number, but they will be all equal to one another.

6. Diameter is any ſtrait line drawn within the circle

circle through the center, from one ſide of the circle to the oppoſite, as A D, and E B.

7. Chord is any ſtrait line drawn from one part of the circumference to the other, ſo it does not paſs through the center as B R, R T, T E.

8. Arch is any part of the circumference.

9. A degree is the 360th part of the circumference, for every circle may be conceived to be divided into 360 equal parts, as in the figure, the inner contains as many parts as the outer one does; but the degrees are greater or leſs in proportion to the radiuſſes of the circles.

A ſemi-circle is half a circle, and contains 180 degrees; a quadrant is $\frac{1}{4}$ of a circle, that is 90 degrees; what an arch wants of 90 degrees is called the complement of that arch; 40° is the complement of 50°, and 50 the complement of 40°. The ſupplement of an arch is what it wants of 180 degrees, 140 is the ſupplement of 40°.

11. When two lines meet at one point they form what is called an angle, the lines that form it are called the ſides or legs, the point where they meet is called the angular point. *Pl.* I. *Fig.* 3.

The quantity of an angle is not eſtimated by the length of the ſides that form it, but by the number of degrees it contains, for let there be two angles A B C, and D E F, and one arch deſcribed from B, and another from E as centers, and likewiſe the two radiuſſes equal; if then the two arches be equal the angles will likewiſe be equal, otherwiſe not. We uſe three letters to expreſs an angle, the middle one denoting the angular point.

12. A perpendicular is a ſtrait line, ſo drawn to another, as to incline to neither ſide, as D C to A B, and if it is not at the end of a line, will make two equal angles, and therefore 90 degrees

grees each; they are called right angles. When the line is not perpendicular as C F, it will incline toward A, and the angles will become unequal, and are called oblique; the angle A C F which is leſs than a right one, is acute; the angle F C B greater than a right one is called obtuſe. *Pl.* I. *Fig.* 4.

13. Parallel lines are ſuch as are every where equally diſtant from one another, as A B, and F G. *Pl.* I. *Fig.* 1.

14. A plain rectelineal triangle is a figure, or a ſpace, upon a plane limitted by three ſtrait lines, which are called ſides, and by their meeting form three angles; if the ſides be equal it is called equilateral: If one of the angles be right, that is 90 deg. it is called a right angled triangle. As it is only of theſe we ſhall treat, when we mention the word triangle, it muſt be underſtood of a ſtrait lined right angled one, unleſs the contrary be expreſſed. The ſides of a right angled triangle are diſtinguiſhed by different names, which we ſhall call their proper names; for we ſhall hereafter have occaſion to give them ſir-names.

15. Hypothenuſe, is the ſide oppoſite to the right angle, which we ſhall call A C. The two ſides that form the right angle, are called the one the perpendicular, and the other the baſe indifferently, but for diſtinction we ſhall call A B the perpendicular, and draw it parallel to the margin of the book; B C we ſhall call the baſe, and draw it parallel to the top and bottom of the page, ſo the angle at B will be 90 deg. the angles at A and C both together will be 90 deg. as ſhall be proved hereafter, the angle at A we ſhall call ſimply the angle, which we ſhall always ſuppoſe oppoſite to the baſe. The angle at C we ſhall call the complement

angle

angle. The following are ſuch natural inferences from the definitions, that they require no demnoſtration. *Pl.* I. *Fig.* 5.

INFERENCES.

1. The greateſt angle that can be made by two ſtrait lines will be leſs than 180 degrees.

2. If a right line C F, meets another line A B any where as at C, ſo as to form two angles, both together will make 180 degrees: For, if from the angular point C, a ſemi-circle be deſcribed, it will exactly meaſure both the angles. All the angles that can be made at a point on one ſide of a line will make 180 degrees; but if they croſs in one point, they will make 360 degrees. *Pl.* I. *Fig.* 4.

3. If there are ſeveral equal chords, as E F, F G, in the ſame circle, the arches ſubtended by them will alſo be equal, but if the circles be unequal, the arches may contain the ſame number of degrees in each, but the chords will be unequal in proportion to the radiuſſes; that is, if the radius C E be double or triple the radius C N; then will the chord E F, be double or triple the chord N M: Though this may ſeem plain enough by the figure, yet as upon it the whole of trigonometry chiefly depends, it ſhall be demonſtrated in Theorem VI. It is likewiſe very plain, that though the arch E F G be equal to the two arches E F, and F G, taken together, yet the chord G E is not equal to the chords E F, and F G. *Pl.* I. *Fig.* 2.

SECT. II. Geometrical Theorems.

THEOREM I.

If two ſtrait lines A B and D E croſs one another in C, the oppoſite angles A C D, and B C E, will be equal; for let the angle D C B, be any number

number of degrees ſuppoſe 100; then the angles A C D, and B C E, muſt each be 80 degrees, by Inference 2d, Sect. 1. Chap. 2. *Pl.* I. *Fig.* 6.

THEOREM II.

If a ſtrait line E F cut two parallel lines A B and C D, then

1. The external oppoſite angles A G E and D H F will be equal; as will alſo the angles B G E and C H F; for the two lines being parallel, they may be conſidered as one broad line, and the line E F croſſing it, ſo the angles will be equal by the preceding. *Pl.* I. *Fig.* 7.

2. The internal angle C H E, is equal to the external one A G E, and E H D equal to E G B, for A G E was juſt now proved equal to D H F, equal C H E by preceding; and for the ſame reaſon E G B=(C H F) E H D. *Pl.* I. *Fig.* 7.

Note, By external is underſtood out-ſide; and by internal inſide.

3. If a line C B be drawn to meet two parallels A B and C D, the alternate angles A B C and B C D will be equal, for producing A B to E; D C to F; and C B to H and G; then the angle H B E is equal to the angle A B C, and likewiſe to the angle F C G=B C D, therefore A B C= B C D; and for the ſame reaſon the alternate angles E B C and B C F are equal. *Plate* II. *Fig.* 12.

THEOREM III.

If a perpendicular be drawn from C, the center of a circle, to the chord A B, it will biſſect the chord in F, and the arch in D; and if a perpendicular be erected from the middle of a chord, it will paſs through the center, and therefore if produced will be a diameter: for the dotted lines C A and C B, being equal, alſo D A and D B, the point C the

is equally distant from the points A and B, and for the same reason, the point D is equally distant from the points A and B, so F, or any other point in the line C D, will be equally distant from the points A and B, and therefore A B is bisected in F. *Pl.* I. *Fig.* 8.

THEOREM IV.

If a strait line A B be drawn to touch the circumference in the point B, and the chord B F be drawn, the angle A B F made by the chord and tangent, is measured by half the arch, of which B F is the chord.

DEMONSTRATION.

Draw the radius C E perpendicular to the chord, which will bisect the arch in E, so shall B E be half the arch F E B, evidently the measure of the angle B C E; all then that is to be proved, is, that the angles A B F and E C B are equal: In order to which draw the radius C B, which will be perpendicular to the tangent, draw also the diameter D G parallel to the chord B F. *Plate* I. *Fig.* 9.

The angles A B F and FBC, both together, make 90 degrees; the angles E C B and B C G are likewise 90 degrees; but F B C and B C G are alternates to the parallels F B and D G, and therefore equal (by the 2d The.) and of consequence the angles A B F and B C E must be equal.

To illustrate this by numbers, suppose the arch B E F 80 degrees, the angle B C E must be 40 degrees, and B C G 50: and because the angles F B C and B C G are equal, F B C must be 50 likewise, and of consequence the angle A B F must be 40 equal to B C E, the thing that was to be proved.

THEOREM V.

An angle made by two chords meeting in the circumference is measured by half the opposite arch, that is the angle L H G is measured by half the arch L E G, of which L G is the chord. *Plate* I. *Fig.* 10.

DEMONSTRATION.

Draw the line K M to touch the circle in H. The three angles at H make 180 degrees (by *Inf.* II. *Sect.* 1. *Chap.* II.) the angle G H M is measured by half the arch G P H; and the angle K H L is measured by half the arch H R L (by preceding) therefore the angle L H G is measured by half the arch L E G, for the half of this arch, and the halves of the other two arches, make 180 degrees: hence we may deduce the following useful inferences.

INFERENCES

1. All the three angles of any strait lined triangle make 180 degrees, for any triangle, as we shall shew in Problem 6, may be inscribed within a circle; then each angle will be at the circumference, and the sides will become chords of arches, the halves of which measure their opposite angles, but half the sum of these arches is 180 degrees.

This inference is of great importance in trigonometry, from whence, if one of the oblique angles of a right angled triangle be known, the other is found by subtraction; we shall therefore demonstrate it another way.

Draw the line H M through T, or any of the angular points of the triangle T S N; and let it be parallel to N S, the side opposite to the angle through which the line H M is drawn. The three angles at T make 180 degrees, but the angles M T S, and T S N being alternates are equal by Theorem II, and for the same reason the angles T N S and H T N

H T N are equal, and therefore the angles at N and S, together with the angle N T S, make 180 degrees. *Plate* I. *Fig.* 11.

2. The angle at the circumference of a circle is half that at the center, if both ſtand upon the ſame arch, for this is meaſured by the whole, and that by half the oppoſite arch.

3. An equilateral triangle is alſo equiangular, for being inſcribed within a circle, the ſides will become equal chords, and of conſequence the arches will be equal, that is, 120 degrees each, which will make the angles 60 degrees each.

4. The radius of a circle is equal to the chord of 60 degrees of the ſame circle; for making an equilateral triangle on the radius A C; the angle at C will be 60 degrees, that is to ſay the arch B F A will be 60 degrees, of which A B is the chord, evidently equal to the radius. It is on this account we always take the chord of 60 degrees before we can make or meaſure an angle by a line of chords. *Plate* I. *Fig.* 12.

5. In two triangles, 1ſt if the ſides of the one be reſpectively equal to the ſides of the other; 2d, if two ſides and the included angle in both be equal; 3d, if two ſides and the angle oppoſite to the like ſide be equal; 4th, if two angles and the ſide betwixt them be equal: in all theſe caſes the triangles will be equal in all reſpects.

THEOREM VI.

In any triangle A B C, if a line D E be drawn parallel to any of the ſides, ſuppoſe to B C; then the ſide A B : A C : : A E : A D; and A C : C B : : A D : D E. Alſo B C : B A : : E D : E A; and C B : C A : : D E : D A. *Plate* I. *Fig.* 13.

DEMONSTRATION.

Divide the line A B into any number of equal parts,

parts, ſuppoſe ſix, in the points 12, 24, *&c.* thro' theſe points draw lines parallel to B C, which will divide the line A C into ſix equal parts in the points 15, 30, *&c.* Again, through the points, in the line A C, draw lines parallel to A B, which will divide the line B C into ſix equal parts, in the points 8, 16, *&c.* ſo ſhall the ſide A C be 90, the ſide A B 72, and B C 48. It is evident that

A B : B C : : A E : E D
72 : 48 : : 60 : 40
A C : C B : : A D : D E
90 : 48 : : 75 : 40
B C : B A : : E D : E A
48 : 72 : : 40 : 60
C B : C A : : D E : D A
48 : 90 : : 40 : 75

now wherever the line be drawn, ſuppoſe *r t*, at 51 in the line A B, then will A *t* be $\frac{51}{72}$ of the line A B, and it is plain A *r* will be $\frac{51}{72}$ parts of the line A C, and *r t* $\frac{51}{72}$ of the line B C. *Pl.* I. *Fig.* 13.

INFERENCES.

1. The chords, ſines, tangents, and ſecants of ſimilar arches, are in proportion to their radiuſſes, for the arches, A L E, and B K H, being ſimilar, that is to ſay of the ſame number of degrees, the chords, ſines, tangents, and ſecants, of the one, are evidently ſimilar to thoſe of the other; and C E : E A : : C H : H B, *&c.* *Plate* I. *Fig.* 14.

2. In ſimilar triangles, the ſides containing the equal angles are proportionals; that is, if the angle at A be equal to that at F, the angle at B equal to that at H, then will the angle at C be equal to that at G, and becauſe the angles at A and F are equal, if A *t* be made equal to F H, and A *r* equal to F G, then the triangle A *r t* will be equal to the triangle F G H, and the line *r t* parallel to B C; ſo ſhall A B : A *t*=F G : : A C : C *r*=F H. *Pl.* I. *Fig.* 13.

3. If

3. If two triangles are ſimilar, and the ſides and angles of one of them be known, and one ſide and the angles of the other, or two ſides and one angle be known, then the other two ſides may be found by the rule of three, for let all the ſides of the triangle A B C be given, and let the triangle F G H be ſimilar to it; and the ſide H G given; then to find the ſides F G and F H it will be B C : : C A : : G F : H G and B C : B A : : H G : H F.

We ſhall have occaſion to have recourſe to this inference in trigonometry, for all the parts of a triangle, ſimilar to the triangle concerned, muſt be known, before we can obtain the things required.

We ſhall now proceed to the practical part of geometry, contained in the following problems.

SECT. III. *Geometrical Problems.*

PROBLEM I.

To divide a ſtrait line A B into two equal parts. *Plate* I. *Fig.* IV.

1. With one foot of the compaſſes in A, with any convenient extent deſcribe two ſmall arches, the one above, and the other below the line.

2. With the ſame extent, and one foot in B, deſcribe other two arches to cut the former in D and E, and draw the line D E, which will cut the line A B in two equal parts in C; for it is evident the points D and E are the ſame diſtance from A that they are from B, and the line D E being the neareſt that can be drawn betwixt theſe two points, all the parts of it will be the ſame diſtance from A, they are from B, and of conſequence the point C will be in the middle of the line; in practice there will be no occaſion to draw the line D E, as the point C may be found by laying a ruler from D to E.

PRO-

PROBLEM II.

At the point C, near the middle of a line, to erect a perpendicular.

1. With any convenient extent take two points A and B equally distant from the point C. *Pl* I. *Fig.* 4.

2. Describe one arch from A and another from B to interfect one another in D, fo fhall D C be the perpendicular required.

PROBLEM III.

From the point D to let fall a perpendicular to the line A B near the middle of it.

1. With any convenient extent from D, as center, defcribe two arches to cut the line in any two points, as in A and B. *Plate* I. *Fig.* 4.

2. Defcribe one arch from A, and another from B, with the fame radius, to interfect one another in the oppofite fide of the line, in E; and lay a ruler from D to E, fo fhall D C be the perpendicular required. Thefe two need no demonftration, for the points D and E being the fame diftance from A they are from B, and A and B being equally diftant from C, the line D C inclines no more to one fide than to the other.

PROBLEM IV.

From the point B at the end of the line to erect a perpendicular.

1. Affume any convenient point C out of the line, in which place one foot of the compaffes, and open the other to B. *Plate* I. *Fig.* 15.

2. With the radius C B defcribe one arch to cut the line at A, and another arch oppofite thereto.

3. Lay a ruler over A and C to cut the oppofite arch in D, fo fhall D B be the perpendicular required.

PRO-

PROBLEM V.

To let fall a perpendicular from the point D, when it will be near the end of the line.

This is only the reverſe of the former: for draw a line from the point D, to interſect the line any where as in A, and find C the middle of this line, from which as center, with the radius C D=C A, deſcribe an arch to cut the line in B, ſo ſhall D B be the perpendicular required: for it is plain if the ſemi-circle A B D be deſcribed from the center C, A D will be a diameter, and the angle D B A at the circumference, which will therefore be meaſured by half the oppoſite arch (by Theorem 5.) In this caſe 180 degrees, therefore the angle at B will be 90 degrees. *Plate* I. *Fig.* 15.

PROBLEM VI.

To deſcribe a circle through any three points A B D which are not ſituated in a ſtrait line. *Pl.* I. *Fig.* 16.

1. Draw the chords A B and B D, for they will be chords, becauſe the points muſt be in the circumference.

2. Biſect the chords by two perpendiculars to interſect one anothet in C, which will be the center required.

PROBLEM VII.

To draw a line parallel to a given one A B.

1. If the diſtance betwixt the lines be given; with that, as radius, draw one arch from A, and another from B, and draw the line C K juſt to touch theſe arches, which will be parallel to A B. *Plate* II. *Fig.* 1.

2. If the point N be given, through which the line is to be drawn.

1. Fix

1. Fix one foot of the compaſſes in the given point N; and open the other to cut the given line any where ſuppoſe at B.

2. With that radius, and one foot any where in the line A B, as at M, deſcribe an arch.

3. With M B as radius, from the center N deſcribe an arch to cut the former in K, ſo ſhall K N, be parallel, to A B.

DEMONSTRATION.

Draw the line K B; ſo ſhall the triangles K B N and K B M be equal, for K N=B M and K M= N B by conſtruction, and K B common to both, therefore the alternate angles K B M, and B K N, are equal, and the lines parallel by Theorem II.

PROBLEM VIII.

To make an angle A B C equal to a given one D E F.

Deſcribe an arch from the center E, and one from the center B, both with the ſame radius, which may be aſſumed at pleaſure, and make this laſt arch equal to the former, ſo ſhall the angles be equal by Def. 13. *Plate* I. *Fig.* 3.

By the ſame method we make an angle of any number of degrees, but, as was before obſerved, we muſt firſt divide a circle into degrees by the following method.

1. Deſcribe a circle and quarter it with the two diameters A D and E B, ſo ſhall each quarter be 90 degrees. *Plate* I. *Fig.* 2.

2. Take the radius in the compaſſes, which lay off on the circumference both ways from the points A, B, D, and E, this will divide the quadrant into three equal parts, and therefore 30 degrees each: Again, theſe ſubdivided into three equal parts, will divide the quadrant into nine equal parts, which will be

be ten degrees each, this is ſufficient for our purpoſe, our intention being only to ſhew how the circumference may be divided, which the mathematical inſtrument makers have effected, not only to ſingle degrees, by repeated trials, but even to minutes and ſeconds, by a contrivance which ſhall be explained in another place. After one circle is divided, any other circles may be divided by drawing them from the ſame center, and producing the radiuſſes of the divided circle to the circumferences that are to be divided. *Plate* I. *Fig.* II.

Being now provided with a divided circle, we may make an angle of any number of degrees, which is in effect making an angle equal to a given one; for let it be required to make an angle of 40 degrees, we have the angle H C B in the quadrant A B which is 40 degrees; ſo there is no more to be done, but to take the radius of this, or indeed of any divided circle, and with that deſcribe an arch, the center of which muſt be the point where the angle is to be made, and then take the required number of degrees from the periphery of the divided circle, and ſet them off on the arch. In practice this is done by a line of chords, for it muſt be obſerved, that in meaſuring arches we do not apply an inſtrument to the curve, but take the diſtance, betwixt the extremities of the arch, with a pair of compaſſes, which in effect is taking the chord of the arch. Now, if from the point B lines be drawn to the ſeveral diviſions of the arch B D, they will be chords of the arches 10, 20, *&c.* and B D will be the chord of 90 degrees. We may then from the center B deſcribe arches through the ſeveral diviſions of the quadrant to interſect the line B D in the points 10, 20, *&c.* which will be evidently equal to the reſpective chords in the quadrant; and it is by this method the lines of chords

on the plain ſcale are conſtructed. In order to make an angle by a line of chords: firſt, with the chord of 60, deſcribe an arch, this will be the ſame as if we take the radius of the divided circle, becauſe the chord of 60 degrees is equal to it, by Inference 4, Theorem V. Secondly, take the required number of degrees from the line of chords, which will be the ſame thing as if taken from the circumference, and lay them off on this arch, as ſhall be illuſtrated in the following examples; for it is in making and meaſuring angles, raiſing and letting fall perpendiculars, that the practical part of Geometry is chiefly employed. There are ſeveral lines of equal parts on the plain ſcales, theſe parts may repreſent feet, yards, miles, or any other meaſure, and as all arches of circles, or which is the ſame thing, all angles, are meaſured by a line of chords; ſo are the lines that form the angles meaſured by a line of equal parts; great care muſt therefore be taken not to miſtake the one for the other, as it often happens with beginners. It is preſumed the following queſtions may be ſolved without any further directions than referring to the preceding problems. We ſhall only obſerve that A B is to be drawn parallel to the margin of the book, A C oblique, and B C parallel to the top or bottom. *Plate* I. *Fig.* 5.

QUESTION I. Let it be required to make an angle of 40 degrees at the point A of the line A B, then to make A C 960 miles, and from the point C to let fall a perpendicular to cut the line A B in B′; required the miles in the lines A B′ and B′ C, and the degrees in the angles at B′ and at C?

II. After making the angle at A 40 degrees, as in the preceding, make the line A B 735 feet, and the angle at B 90 degrees, required the angle at C and the length in feet of the lines A C and B C?

III. Erect

III. Erect a perpendicular at the point B, on which lay off 617 leagues from B to C, and then make an angle at C of 50 degrees; how many leagues the lines A C and A B? and how many degrees the angles at A and at B?

IV. After making a right angle at B, make B A 735 feet, and A C 960 feet. Then required the angles at A and at C, and the length of the line B C?

V. Make a right angle at B, as in the preceding, then make B C 617 feet, and C A 960 feet, required the oblique angles; and the line A B?

VI. Make a right angle at B as before, make B A 735, and B C 617 feet required the line A C, and the angles at A and at C?

In the reſolution of theſe queſtions, it is plain a right angled triangle has been conſtructed in each; the angle at B being always 90 degrees; the angles at A and C both together have likewiſe made 90 deg. the angle at C, as was before obſerved, is the complement of that at A; the line A C the hypothenuſe; A B the perpendicular; and B C the baſe. We ſhall find when we come to trigonometry, that theſe queſtions contain all the varieties of right angled triangles.

VII. Let the radius of a circle be 960 feet; how many feet the chord of 40 deg. of that circle?

VIII. Let the radius of a circle be 735 feet, and a chord in that circle 503, how many degrees the arch?

PROBLEM IX.

To divide a ſtrait line, A B, into any number of equal parts, ſuppoſe 10, 100, or 1000. *Plate* II. *Fig.* 2.

1. Divide any line C D into ten equal parts, by a good ſcale of equal parts, or aſſume any quantity for one of theſe parts; and ſet it off ten times from C to D.

2. Make

2. Make C D the ſide of an equilateral triangle, F C D; and the lines F A and F B equal to A B; ſo the lines F D and F C muſt be produced, when C D happens to be ſhorter than A B, which is the line to be divided.

3. Draw lines from the point F, through the ſeveral diviſions of the line C D, which produced will divide the line A B into ten equal parts. Again, to divide it into 100 equal parts, each of theſe muſt be ſubdivided into ten equal parts; therefore take A *x*, which is the tenth part of the line A B, and ſet it from F, to R, and T; then draw the line R T, which will be equal to A *x*, and will be divided into ten equal parts, by the lines already drawn from F; for the triangles F R 1, F C *z*, and F A *x* are ſimilar; therefore F C : C *z* : : F A : A *x*, but C *z* is the tenth part of F C or C D; therefore A *x* muſt be the tenth part of F A or A B, and R 1 the tenth part of R T = A *x*, ſo ſhall R 1 be the hundredth part of the line A B; and if this could admit of being divided into ten equal parts, the line A B would be divided into 1000 equal parts; but here the diviſions would be too ſmall, we muſt therefore have recourſe to the following method:

1. Draw the lines A B, and C D, parallel to one another, and the lines A C, and B D, perpendicular to them.

2. Divide each of theſe four lines into ten equal parts; by the preceding, and draw the parallels and diagonals as in the figure. *Plate* II. *Fig.* 3.

Now in the triangle A C F are nine lines parallel to the baſe, which will conſtitute ten ſimilar triangles; the firſt baſe will be one tenth, the ſecond baſe two tenths, *&c.* of the baſe C F: theſe are called diagonals, by which one inch, half, or a quarter of

of an inch, may be divided into 100 equal parts as thoſe upon the plain ſcales. *Pl.* II. *Fig.* 3.

It may not be improper here to remark, that as by one line of chords any circle may be divided; ſo likewiſe by one good ſcale of equal parts, any ſtrait line may be divided into any propoſed number of equal parts, or into unequal parts, which ſhall be in a given proportion to one another; as for inſtance, if it were required to divide the line A B into five unequal parts, which ſhall have the ſame proportion to one another as the following numbers. 2, 5, 7, 9, 4. *Plate* II. *Fig.* 5.

To do this, draw the line C D, which make 27 by any ſcale of equal parts, and lay off the ſeveral parts as in the figure; then make C D the baſe of an equilateral triangle F C D, and make F E and F G equal to A B, ſo ſhall E G likewiſe be equal to A B, and lines drawn from F to the ſeveral diviſions of the line C D, will divide the line E G into the ſame number of parts and in the ſame proportion with thoſe of the line C D.

We may likewiſe, by a good ſcale of equal parts, meaſure the ſeveral parts of any ſtrait line, provided the length of the whole line be known; as if it were required to find how many equal parts are contained betwixt the figure 1, and the figures 2, 3, 4, *&c.* of the line of numbers on *Gunter*'s ſcales, the whole line from 1 to 1 being 1000. To perform this, *Plate* II. *Fig.* 4.

1. Take the whole line in the compaſſes, with which from the center A deſcribe an arch.

2. Take 10000 off the half-inch diagonal ſcale, which lay on the arch from B to C, and draw the lines A B and A C.

3. Set off the ſeveral diviſions of the line of numbers from the point A, on the lines A B and A C; the figure 1 being at the point A, the diſtances

tances betwixt the ſeveral figures on the line A B, and the ſame figures on the line A C, meaſured on the half-inch ſcale, will be the number of equal parts contained betwixt thoſe figures and 1 at the beginning of the line: and upon examination will be found to be the logarithms of thoſe numbers. *Plate* II. *Fig.* 4.

It is on the ſame principles the Sector is conſtructed, which performs theſe and ſeveral other uſeful operations very expeditiouſly.

SECTION IV.

Of Sines, Tangents, and Secants.

DEFINITIONS.

1. The right ſine of an arch is a perpendicular let fall from one end of the arch to the radius or diameter drawn thro' the other end of it: A D is the right ſine of the arch A B, and is always half the chord of double the arch; for making E N equal to E A, it is evident the ſine A F is half the chord N A. *Pl* II. *Fig.* 6.

2. The co-ſine, or ſine complement of an arch, is equal to that part of the radius intercepted betwixt the right ſine and the center; C D is equal to F A the ſine of the arch E A, the complement of the arch A B; and the verſed ſine of an arch is that part of the radius intercepted betwixt the right ſine and the circumference; D B is the verſed ſine of the arch A B; ſo the co-ſine and verſed ſine both together make the radius.

3. A Tangent to a circle, is a ſtrait line ſo drawn as to touch a circle only in one point, and if to this point be drawn a radius, it will be perpendicular thereto, becauſe it is the ſhorteſt that can be drawn from the center to the circumference; ſo B M is a tangent to the circle in the point B. *Pl.* II. *Fig.* 6.

The

The tangent of any arch, as of A B, is that part of the line B M, intercepted betwixt the point B, at one end of the arch, and the point H, where a line drawn from the center, thro' the point A at the other end of the arch, meets the line B M; ſo B H is the tangent of the arch A B, and E G the co-tangent, or tangent complement, that is the tangent of the arch E A. *Plate* II. *Fig.* 6.

4. The ſecant of an arch, as of A B, is a line drawn from the center, thro' A at one end of the arch, and produced till it meet a perpendicular erected from B at the other end of the arch; ſo C H is the ſecant of the arch A B, and C G the co-ſecant or ſecant-complement of the ſame arch. *Fig.* 6.

The ſine, tangent, and ſecant of an arch, are the ſame with thoſe of the ſupplement; for being drawn according to the preceding definitions, the ſame lines will reſult.

Inferences from theſe Definitions.

1. The ſine of 90 degrees is equal to the radius, for it is half the diameter or chord of 180 degrees; the ſine of 30 degrees is half the radius; or which is the ſame thing, half the chord of 60 degrees.

2. The tangent of 45 degrees is equal to the radius; for let the arch K S be equal to the arch S E, they will be 45 degrees each; R K is the tangent of the arch S K, and is parallel and equal to E C the radius; but the radius cannot be equal to any ſecant. *Plate* II. *Fig.* 6.

3. The radius, ſine, and co-ſine of any arch conſtitute a right angled triangle. In the triangle A C D, right angled at D, the radius C A is the hypothenuſe, the ſine A D is the perpendicular, and the ſine complement C D is the baſe. The radius, tangent, and ſecant of any arch do alſo make a right angled triangle. In the triangle C B H right angled

angled at B, the radius C B is the baſe, the tangent B H the perpendicular, and the ſecant C H the hypothenuſe. *Plate* II. *Fig.* 6.

4. The length of the ſines, tangents, and ſecants, of any arch, cannot be determined, till the length of the radius be fixed, even tho' the number of degrees contained in the arch be known; but having the length of the radius given, ſuppoſe C B, then the ſines, tangents, and ſecants to every degree, and even to every minute, may be drawn, and their lengths meaſured; in the figure it is done every ten degrees, which is ſufficient for our purpoſe, where it is evident, the tangents and ſecants of the degrees of the quadrant of the ſmall circle, are juſt half the tangents and ſecants of thoſe of the great circle, the radius of the one being double that of the other; and it is by this means that the lines on the plain ſcale are conſtructed; firſt actually drawing them to the ſeveral diviſions of the arch, and then transferring them from thence to the ſcale, where there is a line of equal parts adapted to the the radius, which contains 1000 of theſe equal parts. Now when the ſeveral ſines, tangents, and ſecants are meaſured on this ſcale, we may make a table by which the ſines, tangents, and ſecants of every degree may be found by inſpection to a radius of 1000 equal parts. *Plate* III. *Fig.* 1 *and* 2.

Having thus explained the conſtruction of the ſines, tangents, and ſecants, we come now to ſhew that all the varieties of right angled triangles may by them be ſolved.

It was before obſerved, that the ſides of a right angled triangle were diſtinguiſhed by different names, *viz.* hypothenuſe, baſe, and perpendicular; theſe we ſhall call their proper names. It is alſo evident they may likewiſe be conſidered as ſines, tangents, or ſecants, of the arches that meaſure the oblique angles.

Hence

Hence the whole buſineſs of trigonometry conſiſts in finding ſines, tangents, or ſecants; whether the quantity of the arch, and the length of the radius, or the length of the ſine, tangent, or ſecant of the arch be known; or if theſe be known, to find the quantity of the arch: and although all the varieties may be reduced to two caſes, *viz.* one ſide and the angles given, or two ſides and one angle given to find the reſt, yet as they are intended to be applied to navigation, we ſhall divide them into ſix different caſes as uſual, and illuſtrate the whole by an example in each caſe, whereby it will be very plain, that in finding ſines, tangents, or ſecants, to any given arch we make a right angled triangle, as in the following examples.

CASE I. Given the radius A C of 960 feet, required the ſine and co-ſine of an arch of 40 degrees, or which is the ſame thing. In the right angled triangle A B C given the hypothenuſe A C 960 feet, and the angle at A 40 degrees, required A B the perpendicular, and B C the baſe?

1. Draw a line parallel to the margin, and make an angle of 40 degrees at the point A, by a line of chords; (as in Prob. 8.) on which lay off 960 by a ſcale of equal parts from A to C. *Plate* II. *Fig.* 7.

2. Let fall a perpendicular from C, to cut the line parallel to the margin B; ſo ſhall A B (the ſine complement of the angle at A,) be the perpendicular, and B C, (the ſine of the angle at A,) the baſe.

CASE II. Given the radius A B, 735 feet; required the tangent and ſecant of an arch of 40 degrees? Or which is the ſame thing, in the triangle A B C, given the perpendicular A B 735 feet, and the angle at A 40 degrees, required the hypothenuſe and baſe? *Pl.* II. *Fig.* 7.

1. Make the given angle at A as before, and lay off 735, from A to B.

2. Erect a perpendicular at B, to interfect the line A C, in C; ſo ſhall B C, (the tangent of the angle at A,) be the baſe; and A C (the ſecant,) the hypothenuſe.

CASE III. Given the radius B C, 617 feet; required the tangent complement, and ſecant complement of an arch of 40 degrees? or, which is the ſame thing, given the baſe B C, 617 feet, and the angle at A, oppoſite thereto, required the hypothenuſe and perpendicular? *Pl.* II. *Fig.* 7.

1. Erect a perpendicular at B, on which lay off 617, from B to C.

2. Make an angle at C, of 50 degrees, (that is the complement of the given angle) then will A C (the ſecant complement of the angle at A) be the hypothenuſe, and A B (the tangent complement,) the perpendicular.

CASE IV. Given the radius, A C, 960, and the co-ſine of an arch, A B, 735 feet; required the quantity of the arch, and the ſine thereof? Or, which is the ſame thing, given the hypothenuſe A C, 960, and the perpendicular A B, 735 feet, required the angles and baſe? *Pl.* II. *Fig.* 7.

1. Erect a perpendicular at B, and lay off 735 from B to A.

2. Take 960, the given radius, with a pair of compaſſes, and one foot in A, with the other cut the baſe in C, ſo the angles may be meaſured by a line of chords, and the baſe by a line of equal parts.

CASE V. Given the radius A C, 960 feet, and B C, the ſine of an arch, 617; required the quantity of the arch, and co-ſine thereof? Or, which is the ſame thing, given the hypothenuſe A C, 960, and the baſe, B C, 617; required the angles, and perpendicular? This is the ſame with the preceding, only, laying off 617 from B to C, and then taking

960

960 feet in the compaſſes, and one foot in C, with the other cut the perpendicular in A; ſo the angles and perpendicular may be meaſured. *Plate* II. *Fig.* 7.

CASE VI. Given the radius A B, 735 feet, and tangent B C 617; required the quantity of the arch? Or, which is the ſame thing, given the perpendicular A B, 735 feet, and the baſe B C, 617, required the angles, and hypothenuſe? *Pl.* II. *Fig.* 7.

Make a right angle at B, lay off 735 feet from B to A, and 617 from B to C, and draw the line A C, which conſtructs the triangle; ſo the unknown parts may be meaſured.

CHAP. III.

SECT. I. *Trigonometry Arithmetically.*

TO make a table of natural ſines, tangents and ſecants, and thereby to ſolve all the varieties of right angled triangles.

We have, in the preceding chapter, ſhewn how the ſolutions to all the caſes of right angled triangles may be obtained geometrically, but as this would be too tedious for practice, we ſhall here ſhew how they may be very expeditiouſly done by the rule of three.

It is eaſy to obſerve, that, in the conſtruction of all the preceding triangles, though the radius of a circle be given, that is not ſufficient, to obtain the unknownparts, for we muſt always have the radius of another circle given, before we can conſtruct the triangle, *viz.* the chord of 60 degrees, and we alſo have, in the ſcheme for conſtructing the ſines, tangents, and ſecants, a triangle ſimilar to that in queſtion; ſo that in effect we only make a triangle ſimilar to a given one, and when the triangle is conſtructed,

ſtructed, the unknown parts may be meaſured; now all this may be done without conſtructing the triangle.

In order to perform this, we muſt make a table where the ſines, tangents, and ſecants of every degree and minute of the quadrant may be had by inſpection. This has been performed with great accuracy, by ſeveral eminent mathematicians to a radius of 1000000 equal parts; but as in this treatiſe we do not explain the manner of extracting the ſquare root, we ſhall omit the calculations, and ſatisfy ourſelves with meaſuring the ſeveral ſines, tangents, and ſecants that have been geometrically conſtructed, and collecting them as in the following table.

A Table of natural SINES, TANGENTS, *and* SECANTS.

Deg.	Sines.		Tang.		Seca.		
5	871	9962	875	11430	10038	11473	85
10	1736	9848	1763	56713	10154	57588	80
15	2588	9659	2679	37320	10353	38637	75
20	3420	9397	3639	27474	10642	29238	70
25	4220	9063	4663	21445	11034	23662	65
30	5000	8660	5773	17320	11547	20000	60
35	5735	8191	7002	14281	12208	17434	55
40	6428	7660	8390	11917	13054	15557	40
45	7071	7071	10000	10000	14142	14142	45
		Sines.		Tang.		Seca.	Deg

This table gives, by inſpection, the ſines, tangents, and ſecants, of every fifth degree of the quadrant of a circle whoſe radius is 10000. And becauſe it was before proved, that as the radius of any circle, is to the ſines, tangents, and ſecants of the degrees of the ſame circle, ſo is the radius of

any

any other circle, to the ſines, tangents, and ſecants, of the degrees of this laſt circle. Hence it is evident that if the radius of any circle be known, the ſine, tangent, and ſecant of any arch in that circle may be found by the rule of three; or if the radius, and the ſine, tangent, or ſecant, be known, we may, by the ſame rule, find the quantity of the arch. The various ſolutions may be obtained by the two following general proportions.

Caſe I. The radius and arch given to find the ſine, tangent, or ſecant thereof, or, which is the ſame thing, one ſide and the angles, given to find the other two ſides.

As the tabular radius 10000

Is to the tabular ſine, tangent, or ſecant of any arch

So is the radius of any circle, or the given ſide of a triangle

To the required ſine, tangent, or ſecant; or to the required ſide.

Caſe II. The radius, and ſine, or tangent, or ſecant of an unknown arch being given, to find the quantity of the arch? This is the ſame, as if two ſides of a right angled triangle were given, to find the angles. The proportion is

As the given radius, or one of the given ſides,

Is, to the given ſine, or tangent, or ſecant, or to the other given ſide,

So is the tabular radius

To a ſine, tangent, or ſecant in the table.

This laſt number, which is the reſult of the operation, muſt be looked for in its proper column, in the table, and the degrees correſponding thereto, will be the quantity of the arch, or angle required. For, it muſt be obſerved, when an angle is required, we do not work for the angle itſelf, but for the ſine, tangent, or ſecant of it.

The

The following Examples will ſufficiently illuſtrate what has been ſaid on this head, which, it is preſumed, may be performed, either, by multiplication and diviſion, or by the logarithms, without any further directions, than thoſe already delivered in the rule of three, and in the operations the very ſame figures will be found, as in ſome of the examples of that rule.

Caſe I. Given the radius or hypothenuſe 960, and arch 40 degrees, required the ſine and co-ſine thereof? Or, which is the ſame thing, the baſe and perpendicular?

	Logarithms.
As 10000 the tabular radius	4.00000
Is to 6428, the co-ſine of 40 degrees,	3.80807
So is A C, 960, the given radius or hypothenuſe	2.98227
To B C 617.1 the required ſine or baſe	2.79034

Operation. $6428 \times 960 = 6170680$, the logarithm of which is (the ſum of the logarithm of the ſecond and third terms) 6.79034.

As the index is 6, the natural number correſponding thereto, will conſiſt of ſeven figures, and therefore cannot be found in the table, but there will be no occaſion for finding this natural number, becauſe whatever it is, it muſt be divided by the firſt term, the quotient will then be 617.0680; and when the logarithm of the firſt term is ſubtracted from the ſum of the logarithms of the ſecond and third terms, we ſhall have 2.79034 the logarithm of 617.0680, nearly 617.1 and this may be had in the table, becauſe the index is 2; ſo there muſt be but three figures of integers in the natural number, and the fourth figure will be a decimal.

Caſe II. Given the radius, or perpendicular A B,

735 feet, required the tangent and secant of 40 degrees, or the hypothenuse and base?

Case III. Given B C, 617 feet, the radius, or base of the triangle, required the tangent complement, and secant complement, of an angle of 40 degrees, or the perpendicular and hypothenuse?

Case IV. Given A C, 960 feet, the radius or hypothenuse, and A B, 735 feet, the sine complement, or perpendicular; required the angle and sine thereof or base?

Case V. Given A C, 960 feet, the radius or hypothenuse, and B C, 617, the sine of an arch, or the base; required the quantity of the arch, or angle, and the sine complement thereof?

Case VI. Given B A, 735, the radius, or perpendicular, and B C 617, the tangent of an arch or the base; required the quantity of the arch, or angle, and secant thereof?

As these examples have been already done geometrically in the preceding chapter, it will be found when they are done arithmetically, by the same method the first example is done by, we shall find the result agree with the former operations.

SECT. II. *Of placing the three given Terms in proper Order; and where they are to be found.*

In the preceding sections we have supposed the given side to be the radius of a circle; but the solutions may be obtained by making the given side a sine, tangent, or secant.

The proper names of the sides of the triangle are hypothenuse, perpendicular, and base; but as they may be considered as sines, tangents, or secants of the arches that measure the oblique angles, they will thereby acquire another name, which we shall call their sur-name. Now, as any side of the triangle may be made the radius of the arch that measures

meaſures the oblique angles, this will occaſion their ſur-names to vary according to the ſide made radius, for, if the hypothenuſe be made radius, as in Caſe I, the baſe will be the ſine, and the perpendicular the ſine complement of the angle. If the perpendicular be radius, as in Caſe II, the hypothenuſe will be the ſecant, and the baſe the tangent of the angle. If the baſe be made radius, as in Caſe III, the hypothenuſe will be the ſecant complement, and the baſe the tangent complement of the angle. *Note*, By the angle is underſtood that oppoſite to the baſe, denoted by A, and that at C, oppoſite to the perpendicular, is called the complement angle. Before we come to ſhew how to place the given terms, it will be proper to make the two following remarks, which will enable us to diſcover where the given things are to be found.

Rem. 1. When the operation is for a ſide, there is only one of the given things in the triangle, which may be, indifferently, either the ſecond or third term; but we ſhall always make it the third term, and ſo the two firſt terms will be in the table of artificial ſines, *&c.* and the two laſt in the logarithms.

Rem. 2. When the operation is for an angle, there are two of the given things in the triangle, *viz.* the two given ſides; their logarithms, muſt be the firſt two terms. The third term is the radius in the table, whoſe logarithm is, in the table of artificials, 10.000000, and when the logarithm of the fourth term is found by an operation, we muſt look for it in the table of artificials, in its proper column, the degrees correſponding to which is the quantity of the required angle.

Hence it is obvious that the firſt thing to be done before we can ſtate the terms of the proportion, is, to chuſe one of the ſides for the radius, this

this will determine the ſur-names of the other two ſides; ſo that when a ſide is required every caſe will admit of three, and when an angle is required, every caſe will admit of two different operations.

When a ſide is required, the firſt term muſt always be the ſur-name of the given ſide; the ſecond term the ſur-name of the required ſide; the third term the proper name of the given ſide; ſo ſhall the fourth term be the proper name of the required ſide; from whence we have the following general proportion.

As the ſur-name of the given ſide
Is to the ſur-name of the required ſide } theſe to be found in the artificials.

So is the proper name of the given ſide
To the proper name of the required ſide. } theſe to be found in the logarithms.

When an angle is required, there are always two ſides given, and either of them may be made the radius of the arch that meaſures the oblique angles: The firſt term will be the proper name of that given ſide which is made radius: the ſecond term the proper name of the other given ſide; the third term will be the ſur-name of the firſt term, which will always be radius, and the fourth term, the ſur-name of the ſecond ſide; the proportion is

As the proper name of that given ſide, made radius
Is to the proper name of the other given ſide,
So is the ſur-name of the firſt term, *viz.* radius
To the ſur-name of the ſecond term.

Here the logarithms of the firſt two terms muſt be ſought out of the table of logarithms, and when the logarithm of the third, *viz.* 10.000000, taken out of the artificials, is added to that of the ſecond, and the logarithm of the firſt, ſubtracted from

their ſum, we ſhall have the logarithm of a ſine, tangent, or ſecant; this muſt be ſought out of the table of artificials, in its proper column, the degrees anſwering to which will be the angle required.

The different proportions for the ſolution of all the varieties of right angled triangles are expreſſed in the following table.

The proportion for the various ſolutions of the ſix caſes of right angled triangles.

CASE 1. Given hypothenuſe and angle: required perpendicular and baſe?

Radius { H { R: co-ſine :: H: P and R: ſine :: H: B
P { Sect: R :: H: P and Sect: Tang :: H: B
B { Co-ſect: R :: H: B and Co-ſect: Co-tang :: H: P

CASE 2. Given perpendicular and angle: required hypothenuſe and baſe?

Radius { H { Co-ſine: R :: P: H and co-ſine: ſine :: P: B
P { R: Sect. :: P: H and R: T :: P: B
B { Co-tang: co-ſect :: P: H and co-tang: R :: P: B

CASE 3. Given baſe and angle: required hypothenuſe and perpendicular?

Radius { H { Sine: R :: B: H: and ſine: coſine :: B: P
P { T: Sect :: B: H, and T: R :: B: P
B { R: co-ſect :: B: H and R: co-tang :: B: P.

CASE 4. Given hypothenuſe and perpendicular: required angles and baſe?

H: B

Rad. { H { H : P : : R : co-ſine, and R : ſine : : H : B
P { P : H : : R : Sect. and R : T : : P : B

CASE 5. Given hypothenuſe and baſe: required angles and perpendicular?

Radius { H { H : B : : R : Sine and R : co-ſine : : H : P
B { B : H : : R : co-ſect. and R : co-tang : : B : P

CASE 6. Given baſe and perpendicular: required angles and hypothenuſe?

Radius { P { P : B : : R : T and R : ſect : : P : H
B { B : P : : R : co-tang. and R : co-ſect : : B : H

This table contains all the varieties that can happen in the ſolutions of all right angled triangles. It will be convenient, to have the radius in the given things, on account of the cyphers; and it would not be amiſs to work ſome examples all the different ways, and it is preſumed, the ſatisfaction of ſeeing the ſame figures to be the reſult of the different operations, would compenſate the labour. In practice we ſhall have no occaſion for the natural ſines, and therefore they are not in the common navigation books, for if we had them we ſhould work by their logarithms, and theſe we have in the table of artificial ſines, tangents, and ſecants, they being only the logarithms of the natural. The ſine of 5 degrees in the artificials is 8.940296, the neareſt natural number correſponding to which is 8716; but as the index is 8, it ſhould conſiſt of nine figures; which ſhews the radius by which the table was made conſiſted of eleven figures, and whereas the common tables of logarithms go no further

further than four figures in the natural numbers, we took 10000 for the radius by which we made our table of natural ſines, tangents, and ſecants, only to every fifth degree, that being ſufficient to ſhew that the artificials are the logarithms of the natural; for if 6 be ſubtracted from the index of any artificial ſine, tangent, or ſecant, we ſhall have the natural ſine, tangent, or ſecant, correſponding thereto in the table of logarithms; the index of the artificial ſine of 5 degrees is 8, from which taking 6, remains 2.940296; the neareſt natural number correſponding to this is 871.5, which is the natural ſine of 5 degrees to a radius of 10000 as in the table, and all the other natural ſines, tangents, and ſecants, in the table, upon examination, will be found to be only natural numbers, of which the artificial ſines, tangents, and ſecants, are the logarithms.

We have, in Sect. 4, page 16, ſhewn how to conſtruct *Gunter*'s line of numbers, and the manner of working any queſtions in the rule of three by it, and ſhall here ſhew how all the caſes in trigonometry may be ſolved by *Gunter*'s lines. If we work by the natural ſines, tangents, and ſecants, this is performed in the ſame manner as before directed, by the line of numbers only; but as the artificials are all uſed in the calculations, Mr *Gunter* has likewiſe conſtructed a line of artificial ſines and tangents adapted to the line of numbers, which are the logarithms of the natural. Againſt 5 degrees on the line of ſines, is 871 on the line of numbers; againſt 10 on the line of ſines is 1736 on the line of numbers, evidently the natural ſines correſponding to thoſe degrees, as in the table, and it will be the ſame with all the reſt. The lines being thus conſtructed, the operations in triangles will be

the

the ſame as in the rule of three, and may be ſolved by this general rule.

1. Place the terms as before directed, ſo ſhall the firſt and ſecond be of one name, and the third and fourth likewiſe of one name, though different from the two firſt terms.

2. Extend from the firſt to the ſecond term, upon the line of numbers, when they are ſides of the triangle, but, if the firſt two terms be ſines or tangents, we muſt extend on theſe lines, and this extent will reach from the third term, to the fourth. *Note*, If the firſt two are on the line of numbers, the other two will be either on the ſines, or tangents, and the contrary. The two following examples will ſufficiently illuſtrate what has been ſaid on this head.

Example 1. Hypothenuſe 960, angle 40 degrees, required baſe and perpendicular?

In order to make the firſt and ſecond term of one name, inſtead of the radius we ſhall take the ſine of 90 degrees, ſo it will be ſine 90: ſine 40 :: 960: 617 baſe; and ſine 90: co-ſine 40 :: 960: 735 the perpendicular. The extent upon the line of ſines from 90 to 40, will reach on the line of numbers from 960 to 617, and the extent from 90 to 50, (the complement of 40) on the ſines, will reach from 960, on the line of numbers, to 735.

Example 2. Baſe 617, perpendicular 735 feet; required the angle?

735: 617 :: R: T. Here the firſt and ſecond terms are of one name, and to make the third and fourth of one name, inſtead of the radius, take the tangent of 45, and it will be 735: 617 :: tangent 45: tangent 40; or 617: 735 :: tangent 45: tangent 50. In both theſe proportions, the extent from

from the firſt to the ſecond, on the line of numbers, will be the ſame, and of conſequence, when laid off on the line of tangents, it will reach, in both caſes, from 45 to the ſame point in that line.

The line of tangents on the ſcale is numbered from 1 to 45, which is equal to the radius, and the tangents above 45 are on this line in the ſame points with their complements; that is, the tangent of 46 is in the ſame point with that of 44; the tangent of 40 and 50 are in the ſame point; by this means every point in the line of tangents will ſtand for the tangent and co-tangent of any arch or angle; and, in order to know which of the two is that which is required, obſerve whether the firſt or ſecond term be greateſt; for, if the ſecond be the greateſt, the fourth term will be greater than the third, and therefore more than 45; but, if the ſecond term be leſs than the firſt, the fourth term will be leſs than 45 degrees, as in the preceding example. When 617 is the ſecond term, the fourth will be 40, the tangent of the angle; but when 735 is the ſecond term, the fourth will be 50, the tangent complement of the angle.

If there be a ſecant in the proportion, a ſolution cannot be had by theſe lines, becauſe no ſecant and the radius of the ſame circle can be of equal length; for the radius lies betwixt the center and circumference, whereas the ſecant muſt be produced beyond the circumference.

Having now explained all the various ways of ſolving right angled triangles, *viz.* geometrically, arithmetically, and by *Gunter*'s lines, we have ſo far executed the firſt part of the plan. Nevertheleſs it will not be amiſs to recapitulate the ſubſtance of what has been ſaid on the whole.

1. In the rule of three there are four numbers concerned, only two of which are in the triangle; for

for it is a great miſtake to imagine, becauſe in every caſe there are always three parts, independant of one another, of the triangle known, that theſe are the terms of the proportion, no, it is evident, by all the preceding operations, that we have in conſtructing the ſines, *&c.* formed a triangle ſimiliar to that in queſtion, of which we have all the parts, ſo that in effect it is only making a triangle ſimilar to a given one. (*See Theo.* 6, *Chap.* 2; and *Sect.* II *Chap.* III.

2. As the whole of trigonometry conſiſts in meaſuring the ſides and angles of triangles; before this can be performed we muſt actually divide the circumference into degrees, and even into minutes, and from thence conſtruct a line of chords to make or meaſure angles; we muſt likewiſe make a ſcale of equal parts to meaſure the ſides, (*ſee Prob.* 8 and 9) by theſe two lines the geometrical ſolutions are performed, and in order to do the ſame arithmetically we muſt prove

3. That all the angles of any triangle make 180 degrees, that the radius of any circle is equal to the chord of 60 degrees of the ſame circle, (*See Theo.* V. *Inf.* 1 and 4, and that as the radius of any circle is to the chords of the degrees of that circle, ſo is the radius of any other circle to the chords of the degrees of this laſt circle, (*See Theo.* VI. *Inf.* 1. We muſt likewiſe explain the ſines, tangents, and ſecants, and that in different circles they are in proportion to their radiuſſes, and that the ſides of a right angled triangle may be conſidered as ſines, tangents, and ſecants, and a table of natural ſines, tangents, and ſecants calculated, before the ſolutions can be obtained arithmetically. (*See Sect.* IV. *Chap.* II.

4. To prevent the tedious operations by multiplication and diviſion, the nature and uſe of the loga-

logarithms muſt be explained, (*See Sect.* 3, *Chap.* I, alſo the conſtruction and uſe of *Gunter*'s lines. (*See Sect.* 4, page 16.)

We ſhall add one example more in each caſe, and to ſhew that navigation is performed by right angled triangles, we ſhall make uſe of the ſame examples when we come to plain ſailing, only giving the ſides different names.

Caſe 1. Hypothenuſe 180 miles, angle 70° 19′, required perpendicular and baſe?

Caſe 2. Perpendicular 200 miles, angle 33° 45′, required perpendicular and baſe?

Caſe 3. Baſe 60 miles, angle 11° 15′, required hypothenuſe and perpendicular.

Caſe 4. Hypothenuſe 180, perpendicular 61 miles, required angles and baſe?

Caſe 5. Hypothenuſe 240, baſe 133, required angles and perpendicular?

Caſe 6. Perpendicular 302, baſe 60, required angles and hypothenuſe?

To render trigonometry ſubſervient to navigation, the poſitions of places, and their diſtances from one another muſt be determined, which is the buſineſs of geography, the principles of which are next to be conſidered.

CHAP. IV.

Of GEOGRAPHY.

IT has been found by experience that our earth is not a flat extended plane; this the mariner can make no doubt of, for when he is at ſea, out of ſight of land, he finds himſelf in the center of a circle

circle, in the circumference of which the ſea and ſky ſeem to unite the heavens, at the ſame time, forming a concave ſphere over his head. Now, ſuppoſe he ſees any object, ſuch as a ſhip, or ſmall iſland, juſt appearing out of the water, and ſail directly to it, he will find himſelf when he arrives at that place, the ſame diſtance from the circle that terminates his ſight, as before; and likewiſe ſurrounded by a concave ſphere over his head, all the parts of which will be at the ſame diſtance as before, which could not poſſibly be, if the earth and ſea together did not form a body of a globular form. This is what they call the tereſtrial or teraqueous globe, and in order to give us an idea of the poſition of places, they chuſe to lay them down upon an artificial globe, made to repreſent the real one in which we live.

They likewiſe have made another to repreſent the heavens, which they call the cœleſtial globe, for though they form a concave ſphere, yet they have delineated all the conſtellations, and principal ſtars, upon the convex ſuperficies of this globe, in ſuch a manner, that, if it were tranſparent, and the earth in the center, to an inhabitant on the earth, they would all appear as they actually do as the heavens.

SECT. I. *Geographical Definitions.*

Def. 1. A globe, or ſphere, is a ſolid body, which has a point within it equally diſtant from all parts of the circumference; it may be formed by the revolution of a ſemi-circle round its diameter, ſuppoſing the diameter wholly immoveable.

2. The axis of the globe is the line paſſing through the center, round which the artificial globe is turned: The earth likewiſe is ſuppoſed to move round an imaginary axis, which occaſions the diurnal revolution of the ſun and ſtars.

3. The poles are two points in the ſurface of the globe, through which the axis is ſuppoſed to paſs; and if the earth's axis were produced both ways to the heavens, it would paſs through both the cœleſtial poles, round which the heavens, with the ſun, moon, and ſtars, ſeem to move in 24 hours; one of theſe is called the north or arctick; the other, the ſouth or antarctick pole. There is a ſtar in the heavens near the north pole, which, ſeemingly, has little or no motion, and called the north, or pole ſtar.

4. Meridians are great circles interſecting one another in both poles. *Note*, Great circles are ſuch as divide the globe, and of conſequence each other, into two equal parts.

5. The equator, or equinoctial, by mariners called the line, is a great circle equally diſtant from both poles, and therefore biſects all the meridians at right angles.

6. The ecliptick is a great circle in the heavens cutting the equinoctial in an angle of 23° $29'$. The ſun is always in this circle, in which he appears to us to make an entire revolution in one year, this is, what is called the ſun's annual motion; and whereas the heavens ſeem to revolve round us once in 24 hours, the ſun ſeems by that motion to deſcribe a circle in the heavens every day, which is called the ſun's diurnal motion.

7. Parallel circles, are ſmall or leſſer circles, which divide the globe into two unequal parts, thoſe that are drawn upon the globe are parallel to the equator, and are called parallels of latitude on the terreſtrial, but parallels of declination on the cœleſtial globe.

Theſe are all the circles which are actually drawn upon the globes, but there are other imaginary ones, which, though they cannot be actually drawn, becauſe

becauſe they vary as the obſerver changes his ſituation; yet they may be repreſented by the wooden frame, and other apurtenances.

8. The horizon is that circle in the heavens which terminates our ſight; as, ſuppoſing the obſerver at ſea, he always finds himſelf in the center of a circle, the ſea and ſky ſeeming to unite at the utmoſt viſible extent of his ſight. This is repreſented by the wooden frame.

9. The zenith, is that point in the heavens right over the obſerver's head; and that point in the heavens right oppoſite to it, is called the nadir: If a line were drawn from the zenith to the nadir it would paſs through the center of the earth perpendicular to the horizon, and likewiſe paſs through the center of the horizon; ſo the zenith and nadir are exactly 90 degrees from every point in the periphery of the horizon, this is called the rational horizon, and divides the earth into two equal parts, called the upper and the lower hemiſpheres: The horizon ſeen by us is the ſenſible or viſible, and is always parallel to the rational, the ſemi-diameter of the earth being the diſtance betwixt them.

10. Azimuths are great circles paſſing through the zenith and nadir, and therefore perpendicular to the horizon, they, together with the horizon, zenith, and nadir, alter their poſition according to the ſituation of the obſerver, and are, in reſpect to the horizon, what meridians are to the equator, for if the poles be in the zenith and nadir, the equator will become the horizon, and all the meridians azimuths. The meridian of the place is an azimuth circle cutting the horizon in the north and ſouth points, and the azimuth circle, which cuts the horizon in the eaſt and weſt points, is called the prime vertical.

11. Almicanters are leſſer circles parallel to the horizon

horizon, they are likewiſe called parallels of altitude.

12. Latitude of a place is its diſtance from the equator meaſured on an arch of the meridian; it is either ſouth or north, as it lies to the ſouthward or northward of the equator.

13. Longitude of a place, is the diſtance betwixt the firſt meridian, and the meridian of the place meaſured on the equator. In order to determine the latitude and longitude of places, one of the meridians is graduated both ways from the equator to each pole, this is called the firſt meridian; it has no longitude, becauſe the longitude is counted from it; there are parallels of latitude drawn thro' every tenth degree of it, on both ſides of the equator. The equator is divided into 360 degrees, and meridians drawn through every tenth degree of it; and though there be no more actually drawn upon the globe, we may ſuppoſe a meridian and a parallel drawn through any point on its ſurface, and theſe are ſupplied by the brazen meridian in which the globe turns; now the latitudes are always meaſured on the meridian, and will be the neareſt diſtance of any place to the equator, but the longitude of any place is meaſured on the equator, and will never be the neareſt diſtance to the firſt meridian, except when the place is in the equator.

14. Difference of latitude is the diſtance betwixt the parallels of two places meaſured on the meridian.

15. Difference of longitude is the diſtance betwixt the meridians of two places meaſured on the equator.

16. Departure of any place from any meridian is its diſtance from that meridian, meaſured on the parallel of latitude of the place, which will be always leſs than the difference of longitude if the

place

place has any latitude, becauſe this is meaſured on the equator, whereas the departure is meaſured on the parallel: The departure will indeed contain the ſame number of degrees that the difference of longitude does, but the degrees of the departure will contain fewer miles than thoſe of the difference of longitude. If there be two places A and B, in different latitudes and longitudes, the departure betwixt them cannot be exactly determined; for ſuppoſe A to be in 30 degrees, and B in 40, both of the ſame name. A E the diſtance betwixt A and the meridian of B, meaſured in the parallel of A, will be greater than F B, the diſtance betwixt B and the meridian of A, meaſured in the parallel of B; therefore it is uſual to reckon *a b* for the departure in the parallel of 35 degrees, the middle latitude betwixt the two places: Of this more when applied to navigation. *Plate* IV. *Fig.* 1.

17 Declination of the ſun or a ſtar, is its diſtance from the equinoctial meaſured on a meridian.

18. Tropicks are two ſmaller circles drawn parallel to the equator at 23° 29′ diſtant from it, they limit the ſun's courſe; the northermoſt is called the tropick of *Cancer*, and the ſouthermoſt that of *Capricorn*.

19. Polar circles are 23° 29′ diſtant from each pole, that at the north is called the arctick, and that at the ſouth the antarctick.

All the circles on the terreſtrial globe are likewiſe deſcribed on the cœleſtial, for, though the heavens are a concave ſphere, all the circles may be duly repreſented on a convex one, and, if the obſerver be ſuppoſed in the center of the terreſtrial, and both it and the cœleſtial tranſparent, he would ſee all the circles of the terreſtrial globe coincide with thoſe drawn on the convex ſurface of the cœleſtial, as was before obſerved. The meridians would

would interſect one another in the poles in the heavens, and the cœleſtial equinoctial would biſſect all thoſe meridians at right angles in the heavens, in the ſame manner as the equator in the terreſtrial interſects the meridians upon it.

Inference I. The diſtance of the zenith from the equinoctial in the heavens, meaſured on the meridian, is the latitude of the place; for if one line be drawn from the center of the earth to the place itſelf, and another to that place on the earth's ſurface, where the meridian of the place interſects the equator, the angle formed at the center will be meaſured by the arch of the meridian intercepted betwixt the place and the equator, and if both theſe lines be produced to the heavens, the one will terminate in the zenith, and the other in the equinoctial. Now the arch in the heavens contained betwixt the zenith and equinoctial will contain the ſame number of degrees with that intercepted betwixt the place and the equator, which is actually the latitude of the place.

2. If the diſtance of any cœleſtial object from the zenith can be obtained by any inſtrument, and alſo the declination of that object, or its diſtance from the equinoctial, we may then find the latitude of the place, as ſhall be illuſtrated by various examples in another place.

3. The greateſt latitude cannot exceed 90 degrees, and the greateſt difference of latitude cannot exceed 180.

4. If two places be on the ſame ſide of the equator, but, on different parallels of latitude, their difference of latitude will be found by ſubtracting the one from the other; but if they be on different ſides of the equinoctial, their difference of latitude is the ſum of both latitudes added together.

5. In ſailing towards the equator we decreaſe, but

but in ſailing from the equator we increaſe the latitude, ſo that if the latitude ſailed from be known, ſuppoſe A, in 50 degrees north, and the difference of latitude be likewiſe known, ſuppoſe 600 miles, that is 10 degrees ſoutherly, here we are ſailing towards the equator, therefore ſubtracting ten degrees, the difference of latitude from 50 degrees, we have 40 degrees the latitude of B; but if the difference of latitude were northerly, it muſt be added to 50 degrees, which gives 60 degrees the latitude come to. When the difference of latitude exceeds that ſailed from, it is plain we croſs the equator, and come to a contrary latitude, which is found by ſubtracting the latitude ſailed from, out of the difference of latitude; for let that ſailed from be 20 degrees north, and the difference of latitude 30 degrees ſoutherly, the latitude come to would be 10 degrees ſouth.

6. The height of the pole above the horizon is equal to the latitude of the place, for it is evidently equal to the diſtance of the equator from the zenith.

Though the earth is a ſphere, and all the places on its ſurface moſt naturally deſcribed on a globe, it is more convenient, eſpecially for navigation, to deſcribe them on planes; this is what is called the projection of the ſphere, which is the next thing to be done.

SECT. II.

The projection of the Sphere on a Plane.

There are ſeveral ways of projecting ſolids upon a plane, we ſhall only treat of the orthographic, becauſe all that is neceſſary in navigation is in this projection, performed by ſtrait lined right angled triangles.

The manner of projecting any ſolid orthographically

phically on a plane, is by ſuppoſing the body to be cut into two or ſeveral parts by one or more planes parallel to one another; now, if a plane cut the body into two parts, the plane may be ſo far extended that each part of the ſurface may be repreſented on this plane, by letting fall perpendiculars from each point of the ſurface to the plane; theſe lines are called the projecting lines, and will be all parallel to one another; the point where theſe lines meet the plane of the projection, will give the poſition of theſe places on that plane. *Note*, the thing to be projected is called the original, and the plane on which it is to be projected, the plane of the projection.

Let us then ſuppoſe the globe to be cut by a plane paſſing through both poles, it will likewiſe paſs through the center, and of conſequence divide it into two equal parts, each being one half of the globe: Being thus cut it will lay flat on a table; the bottom on which it ſtands will be a circle, in this caſe a meridian, whoſe diameter will be equal to that of the globe. After being thus cut and laid flat upon a plane, if perpendiculars be let fall from all places on its ſurface to the plane on which it ſtands, we ſhall have one half of the globe projected, all within the circle, and, as it is by the latitudes and longitudes the ſituation of places are determined, we may project all the meridians and parallels on this plane, and the places may be laid down according to their true latitudes and longitudes; now all this may be performed with greater exactneſs, without cutting the globe, by the following method; it was neceſſary to obſerve this, to underſtand the principles from which the operations are deduced.

1. Deſcribe the circle P E S Q, to repreſent the meridian; let P be the north, S the ſouth pole; P S the axis, E Q the equator, for though it is only the

the diameter of it, yet, as the plane on which half of the globe now ſtands is a meridian, the half of the equator would be on the ſurface, and if perpendiculars were let fall from each point of it, they would all fall on the line E Q, and for the ſame reaſon all the parallels of latitude will be repreſented by their diameters, and drawn parallel to the line E Q.

2. Graduate the equator into degrees, by laying a ruler over the ſeveral diviſions of the circumference, parallel to the line P S; the points where the ruler interſects the line E Q will graduate it into degrees, and, though they are equal on the globe, they will here be unequal, for they are the ſines of the arches.

The meridians are all circles on the globe equal to the equator, but as they interſect one another in the poles, they cannot be repreſented by ſtrait lines on the plane of the meridian; they will all become curves when projected on this plane, but not circles. One of them which is at right angles to the plane of the projection is a ſtrait line, and here repreſented by the line P S, the earth's axis: In order then to find the points in each parallel through which the meridians muſt paſs, the parallels muſt all be divided into degrees; now the parallels here are repreſented by their diameters, as well as the equator by its diameter, they muſt therefore be divided into the ſame number of parts, and into the ſame proportion with the diameter of the equator, already properly divided; and to divide the diameter of each parallel into the ſame proportion, let us (by *Prob.* 9, *Chap.* II.) make the radius of the equator the ſide of an equilateral triangle A B C, ſo ſhall each ſide of the triangle be equal to the radius of the equator. Then lay off the ſeveral parts of the radius of the equator, upon the lines

A B, and A C; ſo ſhall C, 10, C, 20, in the plane of the projection, be the ſame with A, 80, A, 70, in the line A B, and in the line A C, the ſides of the triangle. Now when lines are drawn from the points 10, 20, *&c.* in the line A B, to the ſame points in the line A C, they will be equal to the radiuſſes of the ſeveral parallels, and all parallel to B C, the baſe of the triangle; all that remains to be done is to transfer the diviſions of the radius of the equator to B C, the baſe of the triangle, and then lines drawn from A to the ſeveral diviſions of the baſe, will divide all the parallels into the ſame proportion with the equator.

Having found all the points, the meridians may, by a ſteady hand, be drawn through theſe points, for as they are elipſes they cannot be drawn with the common compaſſes, which is an objection to this projection on the plane of the meridian; but this obſtacle is entirely removed, becauſe, in the caſes we ſhall make uſe of it, we ſhall have no occaſion to draw the curves, only to find the place in any parallel of latitude where any aſſigned meridian ſhall interſect it; or, if that be known, to find where the meridian will interſect the equator. However we ſhall ſhew how the meridians may be projected into ſtrait lines by the ſame principles.

Let the globe be cut by a plane paſſing through the equator, then laid flat upon a plane, the ſection will be a circle, and the baſe or bottom on which it ſtands will be the equator, and, when the half of the globe is in this poſition, if a perpendicular be let fall from the pole, it will come right in the center of the circle, and, as all the meridians interſect one another in the pole, they will, in this projection, be ſtrait lines, and the parallels of latitude will be circles: The radiuſſes of each will be the ſame in both projections, that is, the ſines of their

their complements. Let the line A P C be the diameter of the firſt meridian, and P A the radius properly divided into degrees, which will thereby become a line of ſines; and when circles from P, as center, are drawn through the ſeveral diviſions 70, 80, *&c.* they will all be parallel to the equator, and therefore parallels of latitude, and it is evident their radiuſſes will be the ſines of 10, 20, *&c.* the complements of their latitudes in this projection. In the former projection G H is the radius of the parallel of 40 degrees, evidently the ſine of 50 degrees, the complement of the latitude.

The meridians and parallels being thus drawn, places may be laid down according to their latitudes and longitudes taken out of the tables. *Plate* IV. *Fig.* 3.

Let C E be 60 miles, equal a degree of the equator; draw the lines P E and P C, ſo ſhall the diſtance betwixt theſe lines in each parallel be one degree, and when meaſured by the ſame ſcale of equal parts, that the 60 miles in the equator was taken from, we ſhall have the number of miles that make a degree in any of theſe parallels of latitude: This is the very ſame thing, as if the difference of longitude betwixt two places in one parallel of latitude were given, to find their departure, that is, their diſtance in that parallel: And to find it by calculation, the proportion will be, As the radius of the equator is to the radius of the parallel; (or, which is the ſame thing, the ſine-complement of the latitude,) ſo is their difference of longitude to their departure; or, if the departure be given to find the difference of longitude, it will be, As the radius of the parallel (or co-ſine of the latitude) is to the radius of the equator, ſo is the departure (or diſtance betwixt two meridians in any parallel of latitude, to the difference of longitude,

gitude, or the diſtance betwixt the ſame meridians on the equator.

It would ſeem that before a reſolution could be obtained by the preceding proportions, both the earth's ſemi-diameter, and that of the parallel ſhould be given in miles; but this objection is removed by conſidering that, in the rule of three, inſtead of the two firſt terms we may take any other two that have the ſame proportion or ratio. For this will make no alteration in the fourth term, becauſe the ratio of the third and fourth will ſtill be equal to that of the firſt and ſecond; ſo, inſtead of the radius of the equator, we may take the radius of any other circle, as that in the table of ſines, *&c.* and then the ſine complement of the latitude may be taken out of the ſame table. If the two places be in different parallels of latitude, and within 20 degrees of the equator, the departure may be accounted in the parallel of the middle latitude; and, if the difference of latitude does not exceed two or even three degrees, the middle latitude may be uſed ſafely in moſt caſes; then, inſtead of the co-ſine of the latitude, ſay co-ſine of the middle latitude, and in all other reſpects the proportions will be the ſame as before. We ſhall have occaſion to refer to theſe proportions when we come to treat of parallel, or middle latitude ſailing.

Now, though all the places on the globe may be laid down upon the plane, either of the meridian or of the equator, according to their true latitudes and longitudes, and thereby their true diſtances obtained, yet this will not anſwer the purpoſes of navigation, for (the meridians interſecting in a point) any line as an arch of a great circle on the globe, or the projection of it on a plane, whether in a ſtrait or curve line, will croſs all the meridians at unequal angles, whereas the line the ſhip deſcribes by

by the direction of the compaſs makes equal angles with all the meridians, and is called the rhumb line, the angle this line makes with the meridian is called the courſe. Now before this angle can be found, even on land, where the places are in ſight of one another, the meridian muſt be found by ſome means, which is effected, at ſea, by the mariner's compaſs. This inſtrument is only a circle divided into 32 equal parts, called points, each ſubdivided into halves and quarters; and the ſeveral lines which are called rhumbs, drawn from the center to the circumference conſtitute the angles, and are meaſured by the circumference. The meridian on this inſtrument is repreſented by that rhumb which is diſtinguiſhed by the fleur-de-lis. Now, if a ſhip ſails on any other rhumb as ſuppoſe on the South Weſt, the angle will always be meaſured by the arch that lies betwixt the North and the South Weſt points of the compaſs which never varies. So the rhumbs make equal angles with all the meridians, and becauſe this cannot be effected in any projection where the meridians interſect in a point, there muſt be ſome expedient found where the meridians may be repreſented by ſtrait lines, which is the caſe of the ſea charts.

CHAP. V.

Of the SEA CHARTS.

THERE are two ſorts of ſea charts, *viz.* plain and *Mercator*'s, and in both the meridians are parallel to one another, which is a great error; becauſe, in that caſe, a degree in any parallel contains the ſame number of miles that a degree of the

the equator does, for which there is no remedy in the plain chart, but as this chart is abſolutely neceſſary in navigation, we ſhall here ſhew how it may be conſtructed, and afterwards, how the errors in the plain are corrected by *Mercator*'s chart.

SECT. I.

Conſtruction of the plain Chart.

This may be made to contain the whole or any part of the earth, but as the parallels of latitude are equal to the equator, it will be ſufficient for our purpoſe to make one from the parallel of 50, to that of 61. To perform this, there is no occaſion to ſuppoſe the earth to be a flat extended plane, but only to repreſent the equator by a ſtrait line; and becauſe the meridians are all perpendicular to the equator, if they be likewiſe repreſented by ſtrait lines, this will, of neceſſity, occaſion them to be parallel to one another, the thing that is propoſed. *Plate* V. *Fig.* 2.

Draw the line Y Z to repreſent the parallel of 50 degrees, and erect the perpendiculars W Y, and X Z, to repreſent two meridians, and in this it will agree with the globe, becauſe the meridians interſect all the parallels at right angles.

2. By any convenient ſcale of equal parts, lay off 660 from Y to W, and from Z to X, and draw the line W X which will be the parallel of 61 degrees, for 11 degrees is 660 miles.

3. Divide the meridians into degrees; thus, take 60 by any line of equal parts, which lay off on the lines Y W, and Z X, to 51, and from 51 to 52, *&c.* and where the degrees will admit, they may be ſubdivided into halves, and quarters, or into miles.

4. Gra-

4. Graduate the parallels W X, and Y Z, into degrees, making them equal to thoſe of the meridian, and draw the line A V to repreſent the firſt meridian, from whence the longitude is to be accounted.

The chart being thus limited, we may lay down places upon it by their latitudes and longitudes; but it muſt be obſerved that the departure and difference of longitude on this chart are the ſame thing.

Let it then be required to lay down the following places according to their latitudes and longitudes, as follows.

	Latitude North.		Longitude Weſt.	
	Deg.	Min.	Deg.	Min.
B	51	1	2	49
C	54	21	5	02
D	59	23	6	02
E	61	00	6	02
F	59	59	3	13
G	56	39	1	00
H	51	37	0	00
A	50	00	0	00

In order to lay down theſe, firſt draw a line parallel to Y Z, 61 miles diſtant from it. Now it is certain B muſt be ſomewhere in that line, and becauſe it is in Weſt longitude, lay off 169 miles from A and V, towards Y and W, or, which is the ſame thing, 49 miles from the ſecond degree of longitude, and then draw a meridian, which will interſect the parallel in B: In like manner lay off 21 miles from 54 deg. on both the meridians, and there draw another parallel of latitude; then lay off 2 miles on the parallels W X, and Y Z, from the 5th degree

degree of longitude, and draw a meridian to interſect the parallel in C, all the reſt may be laid down by this general rule. Firſt, look for the latitude on both the graduated meridians, where draw a parallel of latitude. Secondly, look for the longitude on the graduated parallels, where draw a meridian, which will interſect the parallel of latitude in the place required: Or, if the places be already laid down, their latitudes and longitudes may be found by drawing a parallel and meridian through each, to interſect the graduated meridians and parallels, which will give their latitudes and longitudes: Or, by taking, with a pair of compaſſes, the neareſt diſtance of the place from any parallel of latitude, and laying it off from that parallel on the graduated meridian, and this will give the latitude: The longitude is found by taking the diſtance of the place from any meridian, and laying it off from the ſame meridian on the graduated parallel.

The places being thus laid down, if we draw a line from A to B, from B to C, *&c.* we ſhall have ſo many right angled triangles. Hence it will be eaſy to apply the doctrine of right angled triangles to navigation, for, it is evident, that upon the plain chart the diſtance, difference of latitude, and departure, always makes a right angled triangle, the diſtance anſwers to the hypothenuſe, the difference of latitude to the perpendicular, and the departure to the baſe, the courſe is the angle which the rhumb line the ſhip ſails upon makes with the meridian, and is obtained by the mariner's compaſs, as was before obſerved. The diſtance is had by the log line, which muſt be very carefully divided, ſo that every knot may contain exactly the 120th part of an hour, for then the ſhip will go juſt as many miles in an hour as there are knots of the line run out in half a minute. It will not be eaſy to determine

mine how many miles will make one degree; this, on land, has been done by actual menſuration, by ſeveral eminent mathematicians, to whom recourſe muſt be had for determining this point; but as the mariner may have frequent opportunities of ſailing on the meridian, if then the obſerved latitude agrees with that by account, it is probable the line is truly divided, but if there be any conſiderable difference, it may be preſumed the fault muſt be in the line, for the glaſs may be adjuſted exactly to 30 ſeconds; and if, upon frequent trials, the ſame error is found, the line may then be truly divided. But, as the courſe cannot be depended upon with certainty on account of currents, it is no wonder the mariner often falls into great errors.

The Reſolution of the Six Caſes of Plain Sailing.

As theſe will be ſo many right angled triangles, we ſhall refer to the ſix caſes in trigonometry for their conſtruction; and here only remark, that, as the angle is given in points of the compaſs, it, for the moſt part, will conſiſt of degrees and odd minutes, which cannot be taken off the line of chords exactly, for which reaſon there is a line of rhumbs to be uſed, inſtead of the chords, for ſetting off the courſe; this is only a line of chords, the quadrant being firſt divided into eight equal arches, and theſe ſubdivided into halves and quarters, and from thence transferred to the chord of 90 degrees, which will be equal to eight points. The line being thus conſtructed, before it can be uſed for the courſe, we muſt firſt deſcribe an arch with the chord of 60 degrees, taken from the line of chords, to which the rhumb line is adapted. Again, as in triangles, the perpendicular is drawn parallel to the margin, it will be convenient to draw the difference of latitude ſo alſo; and of conſequence the

departure will be parallel to the top and bottom, as the baſe is in Trigonometry; for in all books (where it is not otherwiſe expreſſed) eaſt is on the right, and weſt on the left hand, the top north, and the bottom ſouth; ſo that in ſailing betwixt N and W, the angle of the courſe muſt be at the bottom on the right hand; but if between the N. and the E, the angle of the courſe muſt be at bottom, on the left hand: If the courſe be ſoutherly, it muſt be on the top when eaſterly; but on the right hand when weſterly.

In projecting the following examples, it will be proper to draw the equator in each; ſo then we may compare the latitude found by projection with that by calculation. *Pl.* II. *Fig.* 9.

CASE 1. A ſhip in 16 deg. 10 min. N. lat. ſails S W by S $\frac{1}{2}$ W 960 miles; required the latitude come to and departure?

1. Draw the line E Q to repreſent the equator, and becauſe we are ſailing on the S W quarter, erect a perpendicular on the right hand at Q.

2. Lay off, by a ſcale of equal parts, 970 miles, that is 16 degrees 10 minutes from Q to A, which muſt be the latitude, the ſhip ſails from.

3. With the chord of 60 degrees from the center A, deſcribe an arch, on which lay off 3 $\frac{1}{2}$ points, that being the courſe.

4. Draw the line A C, on which lay off 960 miles, the given diſtance from A to C; and, laſtly, let fall a perpendicular from C, to cut the meridian in B; ſo ſhall B C be the departure, A B the difference of latitude, and B Q equal C E, the latitude come to.

CASE II. A ſhip in 3 degrees, 48 min. N lat. ſails N E by N $\frac{1}{2}$ E, till ſhe arrives in the latitude of 16 degrees 10 minutes N, required the diſtance and departure?

This

This is the ſame as the 2d in Trigonometry, and becauſe the courſe is juſt as many points as in the foregoing, and the latitude ſailed from, here, is that come to before, it is plain the triangle will be exactly the ſame as in the preceding; and if we draw D E equal and parallel to A Q, and D A parallel to C B, we ſhall have the triangle properly projected; ſo ſhall A Q, equal D E, be the latitude come to, C A the diſtance, and D A the departure. *Pl.* II. *Fig.* 9.

CASE III. A ſhip in 16 degrees, 10 minutes, N latitude, ſails S W by S $\frac{1}{2}$ W, till her departure is 609 miles; required the diſtance and latitude come to?

This may be conſtructed as the 3d caſe in Trigonometry, but we ſhall here ſhew how it may be done by another method.

1. Draw the equator E Q, meridian Q A, on which lay off the given latitude from Q to A, and make the courſe at A, as in the firſt caſe.

2. Draw A D parallel to E Q, on which lay off 609, the departure, from A to D.

3. Draw a meridian through D, to interſect the rhumb line, drawn from A, in C; laſtly, let fall the perpendicular C B, which will be the given departure, A C the diſtance, A B the difference of latitude, and B Q equal C E, the latitude come to. *Pl.* II. *Fig.* 9.

CASE IV. A ſhip in 4 degrees 16 minutes N latitude, ſails betwixt the S and the W till the diſtance is 960, and difference of latitude 742 miles, required the courſe and departure?

Here having aſſumed A for the place ſailed from, and conſtructed the triangle as in the like caſe of Trigonometry, we may lay off 256 miles, that is 4 degree 16 minutes, from A to Q, ſo ſhall E Q be the equator, and Q B the latitude come to ſouth,

Pl. II.

Pl. II. *Fig.* 7. *Note*, The latitude come to, is found, by taking the latitude ſailed from, out of the difference of latitude, which will always be the caſe, when the difference of latitude exceeds that ſailed from, as here when the ſhip has made 256 difference of latitude ſoutherly, it is plain ſhe comes to the equator, and what more difference of latitude ſhe makes will be in ſouth latitude.

CASE V. A ſhip in 8 degrees 6 minutes, S latitude, ſails betwixt the N and E till her diſtance is 960, and departure 609; required the courſe and latitude come to? *Pl.* II. *Fig.* 7.

This is projected as in Trigonometry, and the equator found as in the preceding, and becauſe the difference of latitude exceeds that ſailed from, we find the latitude come to as in the preceding.

CASE VI. A ſhip in 4 degrees 16 minutes N latitude, ſails betwixt the S and W till her difference of latitude is 742, and departure 609; required the courſe and diſtance? *Pl.* II. *Fig.* 7.

It would be quite needleſs to give any inſtructions for the projecting this, as the triangle is the ſame with the preceding; we ſhall therefore work the firſt caſe arithmetically, but it is to be obſerved that the courſe muſt be converted into degrees before the operation can be performed: Now here the courſe is $3\frac{1}{2}$ points, or 39 degrees 22 minutes.

As radius	10·00000
Is to co-ſine 39° 22′	9.88824
So is diſtant 960 miles	2.98227
To different latitude nearly 742	2.87051
As radius	10.00000
Is to ſine of 39° 22′	9.80228
So is diſtant 960 miles	2.98227
To departure nearly 609	2.78455

In

In the preceding cases, one triangle serves for all; we shall therefore subjoin the following, and, though they may be projected as in Trigonometry, we think it proper here to remark, that the first thing to be done is to draw a meridian; and when there is only one triangle concerned, it is indifferent at what point of the meridian the course be made; when there are more than one triangle concerned, the point in the meridian, for the first course may be assumed, where most convenient, but then this will determine the points for the other courses, in the several meridians. As, for instance, in sailing from A to B, from B to C, *&c.* on different courses; the point A may be any where, so, A B, will be the distance; but, in sailing from B, the point for the course is fixed, therefore we must draw a meridian through B, before we lay off the course, and when C is found we must likewise draw a meridian there, and lay off the course to D, *&c.* This will be best understood by the following examples, which contain all the varieties of plain sailing, and the substance of what is commonly called traverse.

Though all the triangles are already constructed on the plain chart, we shall once more go through the whole process, and, as we intend here to shew the manner of projecting a traverse, make use of the following method.

CASE I. A ship at A, in 50 degrees N latitude, sails W N W $\frac{1}{4}$ W 180 miles to B; required the departure and latitude of B?

1st. With the chord of 60, from the center A, describe a quarter of a circle, or, if need be, a whole or semi-circle, on which lay off the given course, and draw the rhumb line A B.

2dly, Lay off the given distance from A to B.

3dly, Draw a parallel of latitude through B to cut the meridian in 1; so shall A 1 be the

the difference of latitude, and B 1 the departure. *Plate* V. *Fig.* 2.

CASE. II. From the ſame B, ſhe ſails N W by N to C, her difference of latitude is 200; required the diſtance, departure, and latitude of C?

1. Draw a meridian through B; and, in order to make the angle of the courſe, lay off 3 points upon the arch from the meridian of A and draw the line A *x*, and another line parallel thereto, through B. It is plain this will be a N W by N rhumb line.

2dly, Lay off the given latitude, from B to 2, through which point draw a parallel of latitude to interſect the rhumb line in C, ſo ſhall B C be the diſtance, C 2 the departure. *Pl.* V. *Fig.* 2.

CASE III. From C ſhe ſails N by W to D, till her departure is 60 miles; required her diſtance, and latitude of D?—She then ſails 97 miles N to E.

1ſt. Draw a meridian through C, and produce the parallel of latitude C 2, to the left, till C *z* is equal to the departure.

2dly, Draw a meridian through *z*.

3dly, Lay off one point, the given courſe upon the arch, from the meridian of A, and draw the line A *y* and another parallel thereto, through the point C, to interſect the meridian drawn through *z* in D, through which point draw a parallel of latitude to interſect the meridian of C in 3, ſo ſhall C D be the diſtance, C 3 the difference of latitude, and D 3 the given departure. Then lay off 97 miles from D to E, becauſe the courſe is on the meridian. *Pl.* V. *Fig.* 2.

CASE IV. A ſhip at E ſails betwixt the S and E 180 miles, alters her latitude one degree; required the courſe, departure, and latitude, of F, the place come to?

1ſt. Lay

1ſt. Lay off the given difference of latitude from E to 4, through which point draw a parallel of latitude.

2dly, Take 180, the given diſtance with the compaſſes, and placing one foot in E, with the other cut the parallel of latitude in F, ſo ſhall F 4 be the departure, and E F the diſtance, being parallel to A B makes the courſe 6 $\frac{1}{4}$ points, that is S E by E $\frac{1}{4}$ E. *Pl.* V. *Fig.* 2.

CASE V. From F ſhe ſails betwixt the S and E to A, her diſtance is 240, and departure 133 miles; required the courſe and latitude of A?

1ſt. Draw a meridian through F, and produce the parallel of latitude 4 F to the right, till F *v* is equal to the given departure, draw alſo a meridian through *v*.

2dly, Take the diſtance as before, and one foot in F, with the other cut the meridian of *v* in G, through which draw a parallel of latitude to cut the meridian of F in 5, ſo ſhall G 5 be the given departure, F 5 the difference of latitude, and F G the given diſtance parallel to A *x*, therefore the courſe will be S E by S. *Pl.* V. *Fig.* 2.

CASE VI. From G ſhe ſails betwixt the S and E to H, till ſhe alters her latitude 5 degrees 2 minutes, and her departure 60 miles, required the courſe and diſtance and latitude of H? Then ſhe ſails 97 miles S, required the latitude come to, and the departure from the meridian of A?

1ſt. Draw a meridian through G, on which lay off the given difference of latitude to 6.

2dly, draw a parallel of latitude through 6, on which lay off the given departure to H, and draw the line G H, which will be the diſtance, and becauſe it is parallel to A *y*, the courſe will be S by E, and then, becauſe ſhe ſails S, lay off 97 miles on the

the meridian from H, which brings her back to A, the place firſt ſailed from. *Pl.* V. *Fig.* 1.

It is plain when the triangles are conſtructed, the ſhip returns to the ſame place; whereas ſhe will be above a degree to the eaſtward of it, as will appear in the next ſection. Thoſe who incline to amuſe themſelves by projecting traverſes may do it very expeditiouſly by parallel rulers, but they will unavoidably be led into very great errors, for, by this method, of ſolving a traverſe, they cannot find the latitude and longitude the ſhip arrives at, and therefore cannot direct her courſe to the intended port, which is the very thing required. As there are only three different triangles it will be ſufficient to work for the three firſt. In the following proportions we ſhall make the given ſide radius, and as it is all cyphers except the index, it requires no operation, only to ſubtract 10 from the index of the ſum of the logarithms of the firſt and ſecond terms.

CASE I.

Co-fine courſe 70 deg. 19 min.	9.52740
Diſtance 180 miles	2.25527
Different latitude 60.6 miles	1.78267
Sine courſe 70 deg. 19 min.	9.97385
Diſtant 180 miles	2.25527
Departure 169.5	2.22912

CASE II.

Sect-courſe 33 deg. 45 min.	10.08015
Different latitude 200 miles	2.30103
Diſtance 240.4 miles	2.38118
Tang. courſe 33 deg. 45 min.	9.82489
Different latitude 200 miles	2.30103
Departure 133.6 miles	2.12592

CASE

CASE III.

Co-ſect courſe 11 deg. 15 min.	10.70976
Departure 60 miles	1.77815
Diſtant 307.6	2.48791
Co-tang. courſe 11 deg. 15 min.	10.70134
Departure 60 miles	1.77815
Different latitude 301.6	2.47949

It would be altogether needleſs to work for the other three caſes, for they are only the reverſe of the precedings, as will appear when worked by the rules delivered in trigonometry; we ſhall therefore proceed to ſhew how the defects in the plain, may be ſupplied by *Mercator*'s chart.

SECT. III.

The Principles of Mercator's *Chart, by Mr.* Wright's *Meridional Parts.*

Mr *Wright* conſidering that, in the plain chart, each parallel of latitude is enlarged beyond its due meaſure, occaſioned by the meridians being all parallel to one another; and likewiſe that, if the meridians were made to interſect in a point, the rhumb line would make unequal angles with them; which is directly contrary to the angles, the ſhip actually makes on the ſea, in ſteering by the compaſs. He therefore keeps the meridians ſtill parallel to one another as in the plain, and enlarges the degrees of the meridian in the ſame proportion. Here then conſiſts the difference betwixt the two charts. In the plain, the degrees of the meridian are every where equal to one another, and to thoſe of the equator, or (which is the ſame thing) to thoſe of the parallels of latitude. In *Mercator*'s chart, the degrees of the equator are equal to thoſe of the parallels of latitude, but the degrees of the meri-

dian are unequal, being enlarged, as they approach the poles, in the ſame proportion the parallels of latitude are enlarged: ſo the only difficulty will be to graduate the meridians. *Pl.* II. *Fig.* 8.

The firſt thing to be done is to find how many miles will make a degree in any parallel of latitude, the proportion is, as C B, the radius of the equator, is to D A, or C H, the radius of the parallel, or ſine complement of the latitude, ſo is 60, the miles in one degree of the equator, to the miles in one degree of that parallel; this in the parallel of 50 deg. is, by the following operation, 38.17 miles. Now, though this makes one degree in the parallel of 50 degrees on the globe, yet, upon the chart, there are 60 miles in a degree, in that parallel, by which it is enlarged on the chart; therefore the degree of the meridian at that parallel of latitude muſt be likewiſe enlarged in the ſame proportion; ſo that by this method, there muſt always be two proportions, as follows,

1ſt, As radius of the equator	10.000000
Is to the radius of the parallel, or coſine of 50 deg. 30 min. the latitude	9.803510
So is 60, the miles of one degree in the quator,	1.778151
To 38.17, the miles in one degree of the meridian, at the parallel of 50°	1.581661
2d. As 38.17, the miles in a degree in the parallel on the globe,	1.581661
Is to 60, the miles in one degree of the meridian on the globe,	1.778151
So is 60, the miles in a degree in the parallel on the chart,	1.778151
	3.556302
To 94.33, the miles in one degree of the meridian on the chart	1.974641

Now

Now D A, or C H the co-ſine, is to C A the radius, as C B, the radius, is to C E, the ſecant; and D A on the chart, is always equal to D F, or C B, therefore a degree of the enlarged meridian muſt always be equal to the ſecant of the latitude, which may be found by one proportion, as in the following operation.

As radius	10.
Is to the ſecant of 50 deg. 30 min.	10.19649
So is 60, the miles in one degree of the meridian on the globe,	1.77815
To the miles in the meridian betwixt 50 and 51 on the chart	1.97464

The reſult is the ſame, as by the two precedings, and it is by this laſt proportion that Mr *Wright* with great labour and accuracy calculated his table of meridional parts, to every degree and minute of the quadrant from the equator to the pole.

In this table we have, by inſpection, the miles (which he calls the meridional parts) that every parallel of latitude is diſtant from the equator, and of conſequence we have the meridional parts, or the miles that are in the meridian, betwixt any two parallels of latitude by ſubtraction.

As we intend to conſtruct the chart from the parallel of 50 to that of 61 degrees, we ſhall work for the miles in each degree of the meridian, both by the co-ſines and by the ſecants, as in the anexed table, in which there are five columns; the firſt for the degrees of latitude, the ſecond their co-ſines; now, in order to find the miles in a degree of any parallel, the logarithm of 60 muſt be added to that of the co-ſine, and the logarithm of the radius ſubtracted from their ſum, as in the firſt proportion of the preceding operation, the reſult of which is performed in the third column, which will be the

the logarithm of the miles in a degree in that parallel: But there will be no occaſion to look for the natural number correſponding to that logarithm, becauſe, whatever it is, when we come to work for the miles in a degree of the meridian, we muſt ſubtract that logarithm from twice the logarithm of 60, as in the 2d proportion foregoing. Now, as in the third column, we have the logarithm of the miles in a degree of the parallel, ſo, in the fourth, we have the remainder after ſubtracting that logarithm from twice the logarithm of 60; and in the fifth, the natural number correſponding thereto, which is the miles in a degree of the meridian at that parallel of latitude: and this is the reaſon when we are to find the miles betwixt the parallel of 50 and 51, we firſt work for the miles in the parallel of 50 deg. 30 min. which agrees with the operation by the ſecants, and likewiſe with the table of meridional parts. The meridional parts of 50 deg. is 3474.50, the meridional parts of 51 deg. is 3568.83, ſo there will be 94.33 miles betwixt the parallels of 50 and 51, as by calculation; for by the table the parallel of 50 is 3474.5 miles from the equator, to which adding 94.33 we have 3568.83 miles, the diſtance of the parallel of 51 from the equator; and when the ſeveral ſecants to a radius of 60 miles are added together, their ſum will be equal to the meridional parts betwixt the parallel of 50 and of 61.

Latitude		Co-ſines	Sum of coſine, and log. of 60	Remainder	Miles in the meridian.
Deg.	Min.				
50	30	9.80351	1.58166	1.97464	94.3
51	30	9.79415	1.57230	1.98400	96.4
52	30				98.6
53	30				100.9
54	30				103.3
55	30	9.75313	1.53128	2.02502	105.9
45	30				108.8
57	30				111.7
58	30				114.8
59	30				118.2
60	30	9.69234	1.47049	2.08581	121.9
Miles from the parallel of 50 to 61					1174.8

Deg.	Min.	Secants.	Log. of 60 is 1.77815
50	30	10.19649	1.97464
51	30	10.20585	1.98400

Latitude 61 degrees	4649.3	merid. parts.
50	3474.5	
	1174.8	

This table is inſerted to ſhew how the table of meridional parts may be conſtructed; and it is preſumed, our readers may fill up the intermediates which are here omitted, and likewiſe continue the proceſs by the ſecants, which will exactly agree with that by the co-ſines, but, in practice, there will be no occaſion for any of the preceding operations, as the table performs it by ſubtraction, or addition, as before obſerved.

Conſtruction of MERCATOR'*s Chart.*

1. Draw the line Y Z, to repreſent the parallel of 50, and graduate it as before in the plain chart, allowing 60 miles to a degree; erect alſo the two meridians

meridians Y W, and Z X, and A V to repreſent the firſt meridian. *Plate* VI. *Fig.* 1.

2dly, Lay off 94.33 miles, on each meridian, from 50 to 51; 96.39 miles, from 51 to 52; proceed, in the ſame manner, till you come to the parallel of 61, and draw the line, W X, which graduate into degrees as the parallel of 50.

The chart being thus conſtructed, the places may be laid down as in the plain chart; and, as there are two ſea charts, navigation is generally divided into two parts, *viz.* plain and *Mercator*'s ſailing; The firſt we have already explained, and come now to *Mercator*'s.

SECT. II.

Of MERCATOR'S *Sailing.*

In this, ſtrictly ſpeaking, there can be but two caſes; firſt, both latitudes, which muſt always, be given, and either the courſe or departure, thereby to find the difference of longitude: Or, ſecondly, both latitudes and the difference of longitude given, to find the courſe and diſtance; both which are performed by right angled triangles, and may be projected by the two following general rules.

CASE I. Both latitudes, and either the courſe or departure given, to find the difference of longitude.

1ſt, Conſtruct the triangle, as in the plain chart, and produce the diſtance and difference of latitude.

2dly, Find the meridional difference of latitude, which lay off on the proper difference of latitude produced; and then, at that point, draw a line parallel to the departure, to interſect the diſtance produced; which will be the difference of longitude; ſo ſhall the meridional difference of latitude be the perpendicular, and the difference of longitude, the baſe of a right angled triangle.

CASE II.

CASE II; Both latitudes, and longitudes given; to find the courſe and diſtance

1ſt. Make the difference of longitude the baſe, and the meridional difference of latitude the perpendicular of a right angled triangle; ſo ſhall the angle the hypothenuſe makes with the perpendicular be the courſe, which ſuppoſe at the point A.

2. Lay off the proper difference of latitude from A to 1, and draw a line through 1, parallel to the difference of longitude, to interſect the hypothenuſe in B; ſo ſhall A B be the diſtance, and B 1 the departure. *Plate* VI. *Fig.* 1.

The arithmetical ſolutions are obtained by the ſame proportions as in trigonometry.

It was before obſerved in CH. IV. SECT. 2. that the coſine of the latitude is to the radius as the departure is to the difference of longitude; and, it was proved, that the co-ſine of the latitude is to the radius, as the radius is to the ſecant of the latitude, or, which is the ſame thing, as the proper difference of latitude is to the meridional difference of latitude; therefore in finding the difference of longitude, ſay, as the proper difference of latitude is to the departure, ſo is the meridional difference of latitude to the difference of longitude; or, as radius is co-tangent of the courſe, ſo is the meridional difference of latitude, to the difference of longitude.

The difference of longitude may likewiſe be found by ſuppoſing the departure to be made in the middle latitude; the proportion is, As the co-ſine of the middle latitude is to the radius, ſo is the departure to the difference of longitude, and when the different latitude is not above two degrees, it will agree nearly with that by the other proportions.

Note, By departure is to be underſtood the eaſting, or weſting, made in ſailing on one direct courſe.

Though

Though all *Mercator*'s ſailing may be comprehended in the two preceding caſes, we ſhall work for the difference of longitude to each of the ſix foregoing caſes in plain ſailing, and ſhall conſtruct the triangles on *Mercator*'s chart as before. *Pl.* VI. *Fig* 1.

Case 1. After conſtructing the triangle, as on the plain chart, the ſhip comes to B: ſo A 1, is the proper difference of latitude; the latitude come to is 51 deg. 1 min; the meridional parts by the table is 3570.4, and the meridional parts of 50 deg. is 3474.5, which makes the meridional difference of latitude 95.9; lay this off on the meridian from A to H; through H draw a parallel of latitude to interſect the diſtance produced in I; ſo ſhall H I, be the difference of longitude.

Case 2. 1ſt, Draw a meridian through I, and find the meridional difference of latitude as before, which lay off from I to *a*; and through *a* draw a parallel of latitude.

2dly, Make the angle of the courſe at I, or which is the ſame thing, through I draw a line parallel to B C, to interſect the parallel of latitude drawn through *a* in K; ſo ſhall *a* K be the difference of longitude.

Case 3. 1ſt. Draw a meridian through K.

2dly, Lay off the meridional difference of latitude from K, to *b*, and there draw a parallel of latitude.

3dly, Lay off the courſe from K, or draw a line parallel to C D, to interſect the parallel of latitude in L; ſo ſhall *b* L, be the difference of longitude, now ſhe is in 61 latitude, therefore draw a meridian through L to M.

Case 4. 1ſt. Draw a meridian through M, on which lay off the proper, and meridional difference of latitude, and draw a parallel of latitude through each.

2dly,

2dly, Produce the diſtance to N; ſo ſhall N *c*, be the difference of longitude. *Pl.* VI. *Fig.* 1.

Case 5. Draw a meridian through N, on which lay off the meridional difference of latitude, from N, to *d*; and there draw a parallel of latitude to interſect the diſtance produced in O; ſo ſhall O *d* be the difference of longitude. *Plate* VI. *Fig.* 1.

Case 6. Draw a meridian through O, on which lay off the meridional difference of latitude from O to *e*, where draw a parallel of latitude to interſect the diſtance produced in P; ſo ſhall P *e* be the difference of longitude; and becauſe ſhe then ſails ſouth to the parallel of 50, draw a meridian through P to Q; ſo ſhall Q be the place the ſhip arives at on *Mercator*'s, and A on the plain chart. Now, to find the diſtance from Q to A; Q A, the difference of longitude on the parallel of 50 is given; therefore, make an angle of 40 degrees with the parallel of latitude at the point Q, becauſe that is the complement of the latitude; ſo ſhall the perpendicular A *f* be the diſtance, or, which is the ſame thing, the departure, for in the triangle Q A *f*, if the hypothenuſe Q A be the radius, then will A *f* be the co-ſine of the latitude; and as radius is to coſine latitude; ſo is Q A, the difference of long. to A *f*, the departure; that is to ſay, the diſtance in the parallel. We have, in the plate, drawn the ſeveral departures parallel to the differences of longitudes, by which the true diſtances may be found on *Mercator*'s chart.

Case 7. Q in 50 degrees, N latitude and 1 deg. 45 min. E longitude; M in 61 deg. N latitude and 9° 59′ W longitude; required the courſe and diſtance from Q to M? *Plate* VI. *Fig.* 1.

1ſt, Find the difference of longitude, by adding the two longitudes together, and the meridional difference of latitude by ſubtracting the meridional

parts of 50 deg. from the meridional parts of 61 find alſo the proper difference of latitude.

2dly, Lay off the meridional difference of latitude from Q to *h*; where draw a parallel of latitude, on which lay off the difference of longitude from *h* to M, and draw the line Q M, ſo ſhall the angle *h* Q M, be the courſe.

3dly, Lay off the proper difference of latitude from Q to *g*, and draw the parallel *g k*, ſo ſhall Q *k* be the diſtance.

We have now projected all the various caſes of plain, and *Mercator*'s ſailing, whereby the errors of the plain chart are conſpicuous; it is alſo evident, they are corrected in *Mercator*'s chart; ſo that both charts are abſolutely neceſſary; and ſtrictly ſpeaking, plain and *Mercator*'s ſailing cannot be ſeparated, the one being imperfect without the other; for, as the difference of longitude cannot be found on the plain, ſo the diſtance cannot be found on *Mercator*'s chart, unleſs the triangle be firſt projected by the principles of the plain chart. We ſhall now calculate for the difference of longitude by three different proportions, to ſhew their agreement with the projections.

Case I. The proper difference of latitude is 61, meridional difference of lat. 95.9, departure 169.5 and the courſe 70 deg. 19 min. the middle latitude 50 deg. 30 min.

As radius		10.
Tangent 70 deg. 19 min.		10. 446452
Meridian difference of latitude	95.9	1. 981819
Difference of longitude	268.1	2. 428271

Proper

Proper difference of latitude	61	1.782672
Departure *(See p. 80.)*	169.5	2.229124
Merid. difference of lat.	95.9	1.981819
		4.210943
Difference of longitude	268.1	2.428271

As co-ſine of midddle latitude 50 deg. 30 min.	9.803510
Is to radius	10.000000
So is the departure 169.5	2.229124
To the difference of longitude 266.5	2.425614

CASE VII. As meridional difference of latitude	1175	3.07004
Is to the difference of long.	704	12.84757
So is radius		10.
To the tangent of the courſe N 30 deg. 56 min. W.		9.77753

As co-ſine courſe 30 deg. 56 min.		9.93337
Is to radius		10.
So is proper difference of latitude 660		12.81954
To diſtance 769.5 miles		2.88617

And by uſing the ſame operations for the reſt, we ſhall find the latitudes and longitudes to be as follows.

	Lat. N.		Lon. W.	
I	51	01	4	28
K	54	21	8	09
L	59	23	9	59
M	61	00	9	59

	Lat. N.		Lon. W.	
N	59	59	4	13
O	55	39	0	02
P	51	37	1	45
Q	50	00	1	45

Having now explained all the varieties of what is commonly called plain and *Mercator*'s ſailing, we ſhall, in the next place, conſider parallel, and mid-

middle latitude, ſailing, which we ſhall likewiſe unite; for they both ſuppoſe the diſtance betwixt the meridians of two places to be given, either in a parallel of latitude, or in the equator; the proportions will be the ſame, as to find how many miles will make a degree in any parallel of latitude; or, having the diſtance betwixt two places in the ſame parallel of latitude to find their difference of longitude. This will admit only of two caſes; and though they have been ſufficiently illuſtrated, in the projection of the ſphere, and alſo on *Mercator*'s chart, we ſhall now treat of parallel and middle latitude ſailing as diſtinct parts of navigation; and explain the principles of each, both geometrically and by calculation.

CASE 1. A ſhip at Q, in the parallel of 50 degrees, ſails weſt to A, and has made 1 deg. 46 min. difference of longitude; required the departure made? or, which is the ſame, the diſtance ſailed from Q to A? *Plate* II. *Fig.* 10.

Here the ſhip makes no difference of latitude, ſo the departure and diſtance are the ſame things. Therefore to conſtruct the triangle,

1ſt. Draw a line parallel to the margin, as in the firſt caſe of plain ſailing, and make an angle of 40 degrees, that is the complement of the latitude at the point Q.

2dly, Lay off 106 miles, the difference of longitude from Q to A; from A let fall a perpendicular to cut the line parallel to the margin in B; ſo ſhall AB, be the departure, or diſtance ſailed.

CASE 2. A ſhip in 50 degrees latitude, ſails weſt 68.3 miles; required the difference of longitude?

This is only the reverſe of the preceding, for having drawn the line A B, make a right angle at B, and lay off the departure 68.3, from B to A; then

then make the angle at A, 50 degrees, that is the latitude, ſo ſhall the angle at Q be the complement of the latitude, and Q A the difference of longitude.

Example in Middle Latitude.

A ſhip in 39 degrees north latitude, and 6 deg. 48 min. weſt longitude, ſails N W by W 216 miles; required the latitude and longitude come to?

1ſt Conſtruct the triangle as in the firſt caſe of plain ſailing, ſo ſhall A B be the difference of latitude, B C, the departure, and C the latitude come to?

2dly, Add the latitude ſailed from to the latitude come to; the half of their ſum will be the middle latitude, and it will be nearly the ſame thing as if ſhe had ſailed weſt, till ſhe had made the whole departure in that parallel; therefore, as in the preceding, make an angle at C of the ſame number of degrees with the middle latitude, ſo ſhall C D be the difference of latitude; and if the meridional difference of latitude be ſet off from A to F, as in the firſt caſe of *Mercator*, we ſhall find F G the difference of longitude, equal to C D. The courſe is 56 deg. 15 min. and the proportions as follows,

R: ſine of 56 deg. 15 min:: diſtance 216: departure 180. R: coſine 56 deg. 15 min.:: diſtance 216: difference of latitude 120.

Latitude ſailed from	39° 00′
Difference of latitude	2 00
Latitude come	41 00
Latitude come to	41° 00′
Latitude ſailed from	39
Sum	80

As

As co-ſine middle latitude 40° 00	9.884254
Is to radius	10.
So is departure 180 miles	12.255272
To difference of longitude 235	2.371018

Latitude.	Meridional parts.
41	2701.6
39	2545.0
Meridional difference of latitude	156.6

As proper difference of latitude	120	2.079181
Is to departure	180	2.255272
So is meridian difference of lat.	156.6	2.194792
		4.450064
To difference of longitude	234.9	2.370883

Having thus gone through the particulars preparatory to Navigation, it remains now to ſhew, how, by a proper application of theſe, the mariner may attain what is intended by this art, *viz.* to direct a ſhip's courſe to any propoſed port.

The firſt thing neceſſary to be known, is, the latitude and longitude both of the port, the ſhip is bound to, and of the place the ſhip is in; then the courſe and diſtance is found by Caſe 7th of *Mercator:* And, being provided with a compaſs, he may ſteer the direct courſe, if the wind will permit, and the log-line will give the diſtance he is to ſail on that courſe. The latitudes and longitudes of places may be had from tables made from actual obſervations, or from charts wherein all the coaſts are laid down; and here we think it will be proper to ſhew how to find the courſes and diſtances on the charts.

SECT.

SECT. III.

Of meaſuring the Diſtances of Places, and finding the Courſe from one to another, upon the Sea-Charts.

Although on *Mercator*'s chart, the degrees of the meridian have the ſame proportion to thoſe of the parallels that they have upon the globe, yet the diſtances cannot be meaſured as they are laid down on the chart: For inſtance, if one place be in 59 deg. 30 min. and another in 60 deg. 30 min. both N, or both S. latitude, and on the ſame meridian, it is plain their diſtance on the globe would only be 60 miles, whereas on the chart it is 120 miles; but on all *Mercator*'s charts there are directions how to find the true diſtances, which will admit of four caſes.

Case I. When the two places are upon the equator, the diſtances are true, for here the degrees are 60 miles each, and the equator is divided into degrees; and where the ſpaces will admit theſe are ſubdivided into minutes, which ſerves for a ſcale for the chart.

Case 2. When the two places are on the ſame meridian, find the latitude of each, the difference of latitude converted into miles, multiplying the degrees by 60, gives their true diſtance.

Case 3. When the places are in different latitudes and longitudes as I K, K L, *&c.* firſt find the difference of latitude as before, and convert it into miles, which call the proper difference of latitude, and take it off the equator.

2dly, Lay a ruler over both places, and take the proper difference of latitude with a pair of compaſſes; put one foot to the edge of the ruler and move it along the edge till the other turned about juſt touches any *E* and *W* line on the chart; then ſtop the

the foot at the edge of the ruler, and open the other foot to the point where the edge of the ruler cuts the parallel; this measured on the equator gives the true distance; that is to say, that which can be sailed by the compass. This is evidently the same as projecting the triangle on *Mercator*'s chart.

Case 4. When the two places are in the same latitude as A and Q in the parallel of 50, or *b* and M in the parallel of 61; here is given their difference of longitude; so their difference may easily be had by calculation, as in the preceding section; but if it be required to measure it on the chart, first take, with a pair of compasses, one degree of the meridian at the parallel, that is from 49 deg. 30 min. to 50 deg. 30 min. if the places be in the parallel of 50 deg. or from 60 deg. 30 min. to 61 deg. 30 min. if the places be in the parallel of 61. Then count how many times that extent of the compasses, which is a degree of the meridian at that parallel, will be contained betwixt the two places, and this multiplied by 60 gives the distance in miles, and if it does not measure it exactly, that is to say, if the distance betwixt the places in the parallel be any number of whole degrees of the meridian, and something more, take that overplus in the compasses and apply it to the graduated meridian, so that one foot of the compasses may be as many miles below, as the other is above the parallel, so shall the miles betwixt the feet of the compasses be the odd minutes, and when added to the former converted into miles, will give the true distance. If it be required to a greater exactness, project a triangle as in the 6th Case of *Mercator* for finding the distance from Q to A. This may be done by laying a ruler over one of the places, in such a manner that the angle it makes with the parallel

parallel, may be the same number of degrees as the complement of the latitude; as for example, suppose two places *r* and *t*, both in 56 deg. 15 min. North latitude, and their departure required: As there are several compasses on every chart, the N E by E rhumb line, makes an angle of 56 deg. 15 min. with all the meridians, and of consequence, an angle of 33 deg. 45 min. with all the parallels, which is the complement of the latitude, so it is only laying the ruler over *r*, parallel to the N E by E rhumb line, and then placing one foot of the compasses in *t*, open the other just to touch the edge of the ruler, which, measured on the equator, will give the true distance, and in this case will reach from *t* to *s*. Upon the plain chart, there is generally a scale of miles or leagues, by which the distances of places may be measured: And as to the bearings, or the courses from place to place, it is only laying a ruler over the two places, and the rhumb line parallel to the edge will give the course.

The mariner having thus found the course and distance to his intended port, either by the tables, or the chart, in proceeding on his voyage he alters his latitude and longitude, and must therefore find the latitude and longitude the ship is in every day at noon; and this is performed by the first Case of *Mercator*'s, for the course is given by the compass, and the distance by the log-line, but then as a ship may alter her course perhaps every hour, it would be an endless labour to project every single course and distance, or to find the latitude and longitude by calculation. In practice, the various courses and distances are every 24 hours reduced into one single course and distance by the table of difference of latitude and departure; and this is what may be properly called working a traverse; for as a ship seldom alters her latitude above two

degrees in 24 hours, there will be no occasion to work for the difference of longitude to every single course and distance; it will be sufficient after they are reduced to one course and distance, to find the difference of longitude for the whole 24 hours, which may likewise be done by this table. And as it is by this table the whole practical part of navigation is performed, we shall here shew how it may be constructed.

SECT. IV.

To make the Table of Difference of Latitude and Departure.

This table gives, by inspection, the difference of latitude and departure to any course from 1 degree to 90, for any distance from 1 mile to 100. Now, if the difference of latitude and departure be calculated for 1 mile, to every degree and quarter point of the compass, we shall have the upper line of all the left hand pages in the table; the rest of the lines are only so many repetitions of the first, which may be found either by addition or multiplication: the second is double the first; the first and second gives the third; the first and third gives the fourth *&c.* All then that is to be done, is to calculate for the first line, which will be only so many different cases of the first case of plain sailing, the course and distance being always given. Now as the distance is always 1, and the course always given, which suppose 28 deg. we shall find the whole may be performed without any calculation, for if we were to work for it, the operation would be

As radius	10.000000
Is to co-sine of 28 deg.	9.945935
So is distance 1 mile	0.000000
To difference of latitude.	

As

As the logarithm of 1 is 0, it is plain when the terms are placed as above, the ſum of the ſecond and third terms, will always be leſs than the firſt term, which ſhews that neither difference of latitude nor departure will be one mile. Let us then ſuppoſe the mile to be divided into 10000 equal parts, we may then find how many of theſe will be in the difference of latitude and departure, by the following operation.

Radius	10.000000
Co-ſine 28 deg.	9.945935
Diſtance 10000	4.000000
Difference of latitude 88.29	3.945935

Now as the firſt and third terms are always cyphers except the indexes, the fourth term will be the ſame as the ſecond, except the index: and as 4 is always the index of the third, and 10 that of the firſt, inſtead of adding 4 to the index of the ſecond term, and ſubtracting 10 from their ſum, we may ſubtract 6 from the index of the ſecond term, as in the preceding operation, 6 from 9 remains 3; the ſame as 4 and 9 is 13; and 10 from 13 remains 3.

Hence the following general rule may be deduced.

1 Look for the ſine and co-ſine of the courſe, either in degrees, or points of the compaſs, in their proper columns, in the table of artificial ſines.

2. Subtract 6 from the indexes, and find the natural numbers correſponding to each in the table of logarithms, which will be the difference of latitude and departure, in ten thouſand parts of a mile; ſo if the courſe be 28 deg. and the diſtance one mile, the difference of latitude will be 0.8829, and the departure 0.4695; but as the table goes no nearer than tenths of a mile, if it is leſs than half a tenth it is not regarded; and when more than a half, it

is accounted one tenth more than is expreſſed by the firſt figure; ſo in the table you will find if the courſe be 28 deg. and diſtance 1 mile, the difference of latitude will be .9, and departure .5.

If the courſe is 1 degree, and diſtance 1 mile, the ſine of the courſe in the artificials is 8.241855; ſubtracting 6 from the index, there remains 2, the natural number correſponding to which is 174.5, and this, being leſs than one tenth part of a mile, it is rejected, and therefore in the table there is no departure; againſt the co-ſine of 1 deg. in the artificials is 9.999934; the proper index after ſubtracting 6 will be 3, the natural number correſponding to which is 9998 ten thouſand parts of a mile, which is in effect, 1 mile as in the table.

Having thus found the differences of latitudes and departures for 1 mile, from 1 degree to 45, we have them likewiſe without any calculation, from 45 to 90; for if the courſe be 45 deg. the complement is alſo 45, which makes the ſine and co-ſine the ſame thing, and of conſequence the difference of latitude and departure are equal; if the courſe be above 45, ſuppoſe 62, its complement is 28, of which we have the difference of latitude and departure as before; now the co-ſine of the courſe is always the difference of latitude, and the ſine of the courſe is always the departure; hence the differences of latitudes to all under 45, will be the departures to their complements to 90, as in the annexed table.

Diff.	28 Degrees.		2¼ points.		29 Degrees.		2¾ points.	
	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
1	0.8829	0.4695	0.8819	0.4714	0.8746	0.4848	0.8579	0.5141
2	1.7658	0.9390	1.7638	0.9428	1.7492	0.9695	1.7158	1.0282
3	2.6487	1.4085	2.6457	1.4142	2.6283	1.4544	2.5737	1.5423
100	88.29	46.95	88.19	47.14	87.46	48.48	85.79	51.41
	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.
	62 Degrees.		5¼ points.		62 Degrees.		5¼ points.	

By

By comparing the above with the table of difference of latitude and departure, in the books of Navigation, they will be found to agree exactly. It would be quite unneceſſary to calculate for any more, this being ſufficient to demonſtrate the principles on which the table is conſtructed.

The table being thus conſtructed, all the caſes of trigonometry, where the hypothenuſe does not exceed 100, may be done by inſpection, which is the method always uſed in working a day's work; and becauſe the diſtances to the ſeveral courſes, for the moſt part, ſeldom exceed 100, the error in 24 hours will not be one tenth part of a mile; but where the diſtances are larger, they may be taken off in parts. We ſhall illuſtrate this by working the ſix preceding examples, as in the following table.

Courſes.	Diſt.	N.	S.	E.	W.
W.N.W.$\frac{1}{4}$W.	180	60.6			169.4
N.W. by N.	240	199.5			133.4
N. by W.	308	302.1			60.1
North.	97	97.			
S.S.E.$\frac{1}{4}$E.	180		60.6	169.4	
S.E. by S.	240		199.5	133.4	
S. by E.	308		302.1	60.1	
South.	97		97.		

Lat. in	Mid.Lat.	Diff. Long.	Long. in
° ′	° ′	° ′	° ′
51 1	50 30	4 27	4 27 W.
54 21	52 41	3 41	8 8
59 23	56 52	1 50	9 58
61 00			9 58
59 59	60 30	5 44	4 14
56 39	58 19	4 14	
51 37	54 8	1 42	1 42 E.
50			

In

In the firſt column are all the courſes; in the ſecond their diſtances; and, becauſe they exceed the limit of the table; in the firſt caſe, find the difference of latitude and departure for 100, and likewiſe for 80, their ſum will be the difference of latitude and departure for 180: for the ſecond courſe and diſtance find the difference of latitude and departure for 100, and double it, to which add the difference of latitude and departure for 40, their ſum will be the difference of latitude and departure for 240: in like manner, find the difference of latitude and departure for 100, and treble it, to which add that of 8, ſo ſhall we have the difference of latitude and departure to 308. Having thus found the differences of latitudes, and departures to the ſeveral courſes and diſtances, we find the northings are equal to the ſouthings, and the eaſtings equal to the weſtings; which makes the ſhip return to the ſame port as in plain ſailing. But when we work for the ſeveral differences of longitude, we ſhall find above a degree difference of longitude. To find the ſeveral differences of longitude, we muſt firſt find the middle latitude, and ſet it down in its proper column. Now, as the departure may be made the baſe, and the difference of longitude the hypothenuſe of a right angled triangle, the angle oppoſite to the baſe being the complement of the latitude, as was obſerved in the preceding ſection on middle latitude; if we call the middle latitude a courſe, the hypothenuſe may be called the diſtance, and the baſe the departure; hence the difference of longitude may be found in the table of difference of latitude and departure, by the following general rule: 1ſt, Find the middle latitude in the table, as if it were a courſe ſteered; and becauſe the table does not go to minutes, it will be ſufficient to take the neareſt in whole degrees; or

it

it may happen the middle latitude may be had nearer in points or quarter points of the compaſs. 2*d*, Find the departure in the difference of latitude column, correſponding to the middle latitude now conſidered as courſe, and the diſtance correſponding thereto will be the difference of longitude.

If inſtead of the middle latitude we take its complement, we may look for the departure in the departure column, ſo ſhall the diſtance be the difference of longitude as before, comparing the reſult of the proceſs by the table to thoſe by calculation, we ſhall find the error ſo very inconſiderable that it need not be regarded. Thoſe who incline to the utmoſt nicety, may calculate for the difference of longitude by the meridional parts; but they muſt firſt reduce the various courſes and diſtances every 24 hours to one true courſe and diſtance by the table of difference of latitude and departure, as in the foregoing operations.

SECT. V.

Of working a Day's Work from the Log, and tranſferring it from thence to the Journal.

Every mariner knows what is meant by the *Log-Board*, this muſt be copied every day at noon into a book; it contains all the courſes and diſtances ſailed the preceding 24 hours, and theſe muſt be all reduced into one courſe and diſtance by the foregoing method. This is what is called *working a day's work*, and as in accounts the ſeveral tranſactions are poſted from the day-book, or journal, into a ledger; ſo, in keeping a reckoning at ſea, the reſult of the operations of every day's work is tranſferred from the log-book to the journal, where the tranſactions of a month's cruize may be contained in one page; therefore it will be neceſſary to be very

very particular in the log-book, and to preſerve the operations of every day's work, to which the journal may refer for particulars.

It muſt be obſerved, that the courſe ſteered is not the true courſe the ſhip makes, and therefore the courſe ſet down in the traverſe table muſt be corrected by allowing for variation and lee-way.

The variation may be found by taking the ſun's amplitude, at his apparent riſing or ſetting, or by his azimuth, when above the horizon, as ſhall be explained in the next chapter; but when no obſervation can be made, recourſe may be had to Mr *Mountaine*'s Variation-chart, which he has conſtructed with great accuracy from a collection of near a hundred thouſand obſervations.

There is yet no expedient found for diſcovering the exact lee-way; this muſt therefore be attained by experience; but it will very much aſſiſt the judgment if the ſhip's wake be ſet every time the log is hove; which is practiſed in ſome ſhips, and a column on the log-board reſerved for that purpoſe.

After the variation and lee-way are determined, we muſt make a traverſe-table of ſix columns; in the firſt of which muſt be ſet down the ſeveral courſes corrected, and in the ſecond their correſponding diſtances; we muſt then find by the table, the difference of latitude and departure to every courſe and diſtance, and ſet them down in their proper columns.—Now, if the ſeveral differences of latitude be of one name, *viz.* all north or all ſouth, their ſum will be the whole difference of latitude made the preceding 24 hours; but if they be of different names, the difference of their ſums will be the whole difference of latitude. The ſame is to be underſtood of the departure.

The

The latitude the ſhip is in, is found by ſubſtracting the difference of latitude, from the latitude ſailed from, when ſailing towards the equator; or adding thereto when ſailing from it. In like manner the longitude is found by adding, or ſubſtracting, according as we ſail from or towards the firſt meridian; but the difference of longitude muſt be found by the following rule: as, was before obſerved,

1. Add the latitude at noon to that of the preceding noon.

2. Take half that ſum, which call the *Mid. Lat.*

3. Look for the middle latitude, as if it was a courſe, amongſt the degrees in the table, or the complement of the mid. lat. and find the departure in in its proper column.

4. Find the departure in the latitude column, correſponding to the middle latitude, now ſuppoſed to be a courſe, and the diſtance correſponding thereto will be the difference of longitude.

Having thus found the latitude and longitude the ſhip is in, the courſe and diſtance to any port may be eaſily found, for the difference of latitude and difference of longitude may be had by ſubſtraction, and the departure may likewiſe be found by the following method, *viz.*

1. Add the lat. of the ſhip and port into one ſum.

2. Take half that ſum for the middle latitude.

3. Look for the middle latitude as if it was a courſe amongſt the degrees.

4. Find the difference of longitude in the diſtance column correſponding to the middle latitude converted into a courſe, and the difference of lat. correſponding to that diſtance, will be the departure; ſo that this is only the reverſe of the rule for finding the difference of longitude.

The following example contains all the varieties in a day's work.

H.	K.	F.	Courſes.	Winds.	Lee-Way.	
1	6	3	S. W.	N. E.		
2	7	4				*Lizard* N. b. E. 6 leg. 49° 57′ N. lat. 5° 14′ W. long.
3	7		S. W. b. W.	E. N. E.		Freſh gales and clear weather
4	7	3		N. E. by N.		Cloudy
5	7	4		N.		Wind increaſes, and the ſea riſes from the N.W.
6	7					Single reeft topſails, and handed ſmall ſails
7	6			N. by W.		
8	6			N. W.		Blows hard; in all reefs TS
9	6	3		N.W. by W.	1 2/4	Handed TS a great ſea with rain
10	5	4				
11	4		W. S. W.	N.W.	3	
12	4					
1	3	4			4	Cloudy, no obſer. var. per chart 1¾ points nearly.
2	2	3				Laid to under a mainſail
3	up	W. b. N. of W. b. S.		N. by W.	5	
4						
5						
6	The wind ſhifted to E.					Moderate STS, wore ſhip out all reefs TS
7	2		S. S. E.	E.	1 2/4	Fine clear weather, and briſk gale
8	3	4				Zenith diſtance 28° 06′
9	5	3				Declination 19 54
10	6					
11	7			E. N. E.		Latitude 48 00
12	8					

Courſe:

Course corrected.	Dist. in miles.	N.	S.	E.	W.
S. $\frac{3}{4}$ E.	18		17.8	02.6	
S. W. by S $\frac{1}{4}$ W.	41		32.9		24.4
S. by W. $\frac{3}{4}$ W.	12		11.3		04.0
S. by W. $\frac{1}{4}$ W.	8		07.8		01.9
S. $\frac{1}{4}$ W.	6		06.0		00.3
S. by W. $\frac{1}{4}$ W.	6		05.8		01.5
S. S. E., $\frac{1}{4}$ E.	17		15.4	07.3	
S. E. by S. $\frac{1}{4}$ E.	15		11.1	10.1	32.1
					20.0
Course made good.	Dist.		D. lat.	20.0	Dep.
S. 7° 00′ W.	110		108.1		12.1

Lat. sailed from	49°	57′
D. lat. S.	1	48
Lat. per account at noon	48	09
Sum of lat.	98	06
$\frac{1}{2}$ Do. is the M. lat.	49	03

Mid. lat	dep. in lat. col.	in Dist. Col. is		
49	12	the diff. long.	0°	19′
Long. sailed from			5	14
Long. in at noon			5	33

The ship is supposed to sail from the *Lizard*, and must have sailed S by W to the place she was in at 2 o'clock, because it then bore N by E. The variation is $1\frac{3}{4}$ points westerly, which must be allowed to

to the left hand, and makes the courſe corrected, S. $\frac{3}{4}$ E. 18 miles diſtant; then ſteered S. W. by W. till 8, with a large wind, allowing variation, makes S. W. by S. $\frac{1}{4}$ W. 41 miles; from 8 to 10 ſhe makes 1 $\frac{1}{2}$ points leeway, corrected by variation, is S. by W. $\frac{3}{4}$ W. 12 miles; from 10 to 12 leeway and variation allowed, the courſe corrected is S. by W. $\frac{1}{4}$ W. 8 miles; from 12 to 2, 4 points leeway, with the variation is S. $\frac{1}{4}$ W. 6 miles; from 2 to 7 lay to, the middle point betwixt her falling off and coming up is W. allowing 5 points leeway with the variation makes S. by W. $\frac{1}{4}$ eſtimating her drift 1 $\frac{1}{2}$ miles per hour is 6 miles; from 6 to 10 allow 2 $\frac{1}{2}$ points leeway, but the variation being the contrary way, makes the courſe corrected S. S. E. $\frac{1}{4}$ E. 17 miles; from 10 to 12 goes large, ſo there is no leeway, and allowing variation makes the courſe S. E. by S. $\frac{3}{4}$ E. 15 miles. The courſes being thus corrected, and the difference of latitude and departure to each being ſet down in their proper columns, the ſum of the weſtings is 32.1 from which ſubtracting the ſum of the Eaſtings 20.0 there remains 12.1 the departure Weſt. The ſum of the Southings is 108.1 the difference of latitude, and becauſe theſe are too large for the tables, take their halves, the neareſt to which is 54.6 difference of latitude, and 6.7 departure, the courſe correſponding to which is S. 70 deg. Weſt, diſtance 35, and this doubled makes 110 as per column.

In order to find the latitude, ſubtract the difference of latitude made from that ſailed from, becauſe the ſhip is in North latitude, and ſailing to the ſouthward, which makes the latitude by account 48 deg. 9 min. To find the difference of longitude, the mid. lat. is 49, as to the odd minutes, they need not be regarded, look therefore for 49 degrees in the table, which will be at the bottom of

the

the book, and in the latitude column correſponding thereto, find 12.1 which is the departure, but 12.5 is the neareſt, the diſtance correſponding to it is 19, which is the difference of longitude, and becauſe the ſhip is in weſt longitude, and ſailing to the weſtward, it muſt be added to the longitnde ſailed from, which makes the longitude the ſhip is in 5 deg. 33 min.

Now having the latitude and longitude the ſhip is in, to find the courſe and diſtance to any other port, ſuppoſe *Cape Finiſter*, uſe the following method as in the operation: 1*ſt*, Find the difference of latitude and difference of longitude, which converted into leagues is 98 the difference of latitude, and 89 the difference of longitude. 2*d*, Find the middle latitude, which is 45 deg. 33 min. or 46 deg. neareſt, this will be found at the bottom of the table. 3*d*, Look for 89 (the difference of longitude) in the diſtance column; and in the latitude column correſponding to that diſtance, and 49 deg. you will find 61.8; but in 45 deg. (or which is the ſame thing 4 points) is 62.9, ſo we may call the departure 62 miles, and difference of latitude 98, the halves of which are 49 and 31, the neareſt to theſe in the tables is 49.2 difference of latitude, and 30.7 departure, which makes the bearings of *Cape Finiſter* S. 32 deg. W. diſtance 58 leagues, which doubled is 116.

Ship	lat. 48° 00′ N.		long. 5° 33′ W.		
Cape Finiſter	43 06 N.		10 00 W.		
Diff. lat. ——	4 54—98 leg.		D. long. 4 27—89 leg.		
Sum lat. 91 06	Diff. long.	Dep.	Diff. lat.	Diſt.	Bearings
½ is M. lat. 45 33	89	62	98	116 lgs	S 32 0 W

In

In this day's work, we find an error of 9 miles in the latitude, which is corrected by obſervation; and tho' it is very probable there may likewiſe be an error in the longitude, yet, till the longitude can be found by obſervation at ſea, we muſt take that by account, which, it is preſumed, will be ſafer than correcting by the common rules; for a current ſetting S. E. or S.W. about half a mile in an hour, would make the latitude obſerved agree with that by account; and if we happen to miſtake the current by allowing S. E. when it is S.W. we make an error of 26 miles; whereas, if we truſt to our account, the error will be only 13 miles. We are indeed ſure, that the diſtance by account is too ſhort; but whether that be owing to the line, glaſs, or current, is very uncertain. If the drift and ſetting of the current could, by any expedient, be found, every time the log is hove it might be accounted for, as if it were a courſe and diſtance, and ſet down with the other courſes and diſtances in the traverſe table.

After working each day's work from the log-book, they may from thence be transferred into the journal, the form of which is hereunto annexed.

Journal

Journal on board his Majesty's Ship MAGNANIME, A. D. 1759.

Weeks day.	Months day.	Winds.	Courſe made good.	Diſt. in miles.	Latitude by account.	Latitude by obſervation.	Longitude.	Bearings, &c. of head-lands.	Remarks on board his Majeſty's Ship *Magnanime*.
		N. E.							
		North.							
		N.b.W.							
Wed.	*Nov.* 16	Eaſt.	S. 7° W.	110	48° 9'	48° 0'	5° 33'	Cape *Finiſterre* S. 32° 0' W. diſt. 116 leagues.	At 2 PM the Lizard bore N.b.E. 6 leagues, we had then a freſh gale and clear, at 8 blowing hard, in all reefs, and at 2 AM it blew a mere ſtorm, ſo that we were obliged to lay to till 6. The remaining part moderate and clear.
		E.N.E.							

CHAP VI.

Of finding the Latitude and Variation of the Compaſs by Cœleſtial Obſervation.

WE have, in the preceding chapter, ſhewn how to find the latitude and longitude the ſhip is in every day at noon, and likewiſe, from thence, to find the courſe to any port, which is what was at firſt propoſed to be performed by the whole art of navigation; but, there are two difficulties which ſeem to be inſurmountable; and yet, if not removed, will deſtroy the very foundation of the whole ſtructure. The firſt is, that the circles from whence the latitudes and longitudes are accounted are are only imaginary on the ſurface of the earth, and therefore inviſible; how then is it poſſible to know, when we are upon the equator, or firſt meridian, or what diſtance we are from either?

In anſwer to this, it was obſerved in geography, that all the circles which are drawn on the terreſtrial globe, are likewiſe drawn on the cœleſtial. Now the heavens are viſible, where we actually ſee the ſun and ſtars every 24 hours deſcribing circles. The ſun, when in the equinoctial, which happens twice every year, by the diurnal revolution, either of the earth or ſun, deſcribes the equator in the heavens; and altho' all aſtronomers allow the motion to be in the earth, becauſe they cannot by any machinery exhibit the various appearances of all the planets, ſuppoſing the ſun to move round the earth; yet, as we conſider only the different appearances that the ſun, or fixed ſtars, make to the inhabitants of this earth, we may very ſafely allow the ſun to move round the earth as it actually appears to our ſenſes, which may be exhibited by a ſimple machinery of paſtboard.

SECT.

SECT. I.

To find the Latitude by Obſervation.

Let Z P O Q N S H Æ, be a meridian in the the heavens, P the north and S the ſouth pole. Let A B C D E F G K, be the earth. To an inhabitant any where upon the earth's ſurface ſuppoſe at A; (the whole heavens together with) the ſun and ſtars would appear to move round every 24 hours; the point P ſeemingly immoveable, always appearing to him at the ſame elevation, which muſt therefore be the cœleſtial pole, and P S the axis paſſing through the center of the earth; then will B be the north, and F the ſouth pole of the earth; K D the equator, on the earth, which, if produced to the heavens, would be Æ Q, the equinoctial. The arch K A, on the earth, would be the latitude of A; and Z the zenith: It is evident the arches K A and Z Æ contain the ſame number of degrees; and as the ſpectator moves either towards K or B, he will have a new point in the heavens for his zenith, juſt as many degrees diſtant from Z as he has moved from A. Now let us ſuppoſe the ſun in the equinoctial, or a ſtar viſible in the pole. We may then, by an inſtrument, take the degrees the ſun is diſtant from the zenith, or the degrees the pole is elevated above the horizon, and either of theſe will give us the latitude, for theſe two arches are equal.

Now, though there is no ſtar in the pole, neither is the ſun in the equinoctial but twice in the year, yet as the ſun's declination, and that of ſeveral ſtars is known, the latitude may be had by the following rules; but it is to be obſerved, the ſun's meridian altitude, or zenith diſtance, muſt firſt be obtained by a quadrant, or ſome other inſtrument, and becauſe

the ſun and ſtars in ſome latitudes perform their diurnal revolutions above the horizon, whereby they will be twice on the ſame meridian in 24 hours, once above, and once below the pole, it muſt be known whether they are at their greateſt or leaſt altitude; and likewiſe, whether we are in ſouth or north latitude at the time of obſervation.

CASE I. Latitude and declination both north, and zenith diſtance, ſouth; or both ſouth, and zenith diſtance north, add the declination to the zenith diſtance, the ſum is the latitude.

EXAMPLE 1.				EXAMPLE 2.			
Decl.	22°	18′	N.	Decl.	2°	16′	N.
Z. diſt.	26	20	S.	Z. Diſt.	1	12	S.
Lat.	48	38	N.	Lat.	3	28	N.

CASE 2. Declination and zenith diſtance both of one name, that is both north, or both ſouth, and the latitude of a contrary name, ſubtract the zenith diſtance from the declination, the remainder is the latitude.

EXAMPLE 1.				EXAMPLE 2.			
Z. Diſt.	70°	56′	S.	Z. Diſt.	5°	44′	S.
Declin.	22	18	S.	Declin.	2	16	S.
	48	38	N.	Lat.	3	28	N.

CASE 3. Latitude, declination, and zenith diſtance, all three of one name, this can only happen to thoſe who are within the tropics; and the latitude is found by ſubtracting the zenith diſtance from the declination.

Example. On the 20th of *May*, 1759, the ſun's meridian altitude, 74 deg. 26 min. and at the ſame time to the northward of me; required the latitude?

	90.	Declin. by the table	20 0 N
Merid. Alt.	74 26	Zenith diſt.	15 34 N
Z. Diſt	15 34	Latitude	4 .26 N

In all the preceding examples the ſun is ſuppoſed to be above the pole; but when the ſun or ſtar is below the pole, to the altitude, add the complement of the declination, and their ſum will be the latitude.

Example. Obſerving the upper ſtar of the two laſt in the ſquare of the *Great Bear* to be 18 deg. 34 min. above the horizon; required the latitude? I find by the table, the ſtar's declination is 58 deg. 34 min.; the complement is 31° 26′ N

Altd. 18 34

Lat. 50 00 N

The latitude and declination may likewiſe be had, by obſerving the greateſt and leaſt altitude of any ſtar which makes an entire revolution above the horizon in 24 hours; as in the following example, obſerving the aforeſaid ſtar's zenith diſtance to be 8 deg. 34 min. the complement 81 deg. 26 min. is the altitude.

Leaſt altitude	18° 34′	Greateſt alt.	81 26
Greateſt	81 26	Leaſt	18 34
Sum	100 00	Diff.	62 52
½ ſum is the lat.	50 00	½ is com. dec.	31 26

Demonſtration. Draw the circle, and quarter it as before; and becauſe H is the ſouth point of the horizon, and the ſun to the ſouthward of the zenith, in the 1ſt and ſecond caſe, lay off 26 deg. 20 min. from Z to R, which will be the ſun's place in the heavens at the time of obſervation, and as the ſun has north declination, it is certain the equinoctial muſt be to the ſouthward of the ſun, therefore lay off 22 deg. 18 min. the declination from

from R to Æ; ſo ſhall Z Æ, be the latitude, the ſum of Z R and R Æ; again, in the 2d caſe, 70 deg. 56 min. is the zenith diſtance which muſt be laid off from Z to U, and becauſe the declination is ſouth, the ſun muſt be to the northward of U, therefore laying off 22 deg. 18 min. from U, we ſhall have Æ Z the latitude, which is the difference betwixt the zenith diſtance and declination. In the 3d example, Z *r* is the zenith diſtance, and *r æ* the declination, ſo ſhall *æ* Z be the latitude.

When the ſtar makes an entire revolution above the horizon, the zenith diſtance, at the time it is above the pole, is Z 4 and the altitude, when below the pole, is O 5, and drawing the dotted line which is biſected by the radius C *p*, we ſhall have O *p* the lat. and *p* 4, or *p* 5, the complement of the declination, and it is plain this is half the arch 4 *p* 5 the difference betwixt the greateſt and leaſt altitude. That the elevation of the pole above the horizon is equal to the latitude, may be thus proved; the arch Æ Z, and Z P, make 90 degrees, but the arch Z P and P O are 90 degrees, therefore Æ Z, the latitude, is equal to P O the elevation of the pole.

SECT. II.

Of the Right Aſcenſion of the Sun and Stars.

The aſtronomers have calculated tables of the declination of the ſun, and of all the remarkable ſtars; but if we do not know at what hour the ſtars come to the meridian, we ſhall be at a loſs when to begin to obſerve; to remedy this, the right aſcenſion of the ſun and of all theſe ſtars are likewiſe collected into a table; ſo the hour that any ſtar comes upon the meridian may eaſily be found.

In

In order to give us a clear idea of the right aſcenſion of the ſun or of the ſtars, it will be neceſſary to obſerve, that their apparent motion is from Eaſt to Weſt; the ſtars riſe and ſet all the year, at leaſt nearly, in the ſame points of the horizon, ſo their declinations may be allowed to continue the ſame, and of conſequence may be ſuppoſed fixed in the heavens, on each ſide of the equinoctial; and the whole heavens, together with the equinoctial, ſun and ſtars, performing an entire revolution in 24 hours. The ſtars will return to the meridian in the ſame ſpace of time they performed their revolution the preceding 24 hours; but this is not the caſe with the ſun, for he has an apparent annual as well as diurnal motion; ſo that he cannot, like the ſtars, be a fixed point in the heavens, elſe he would like them riſe and ſet always in the ſame point of the horizon; and that it is not ſo, is evident to every man's obſervation, without the aſſiſtance of any inſtruments.

The ſun's annual motion is from Weſt to Eaſt, in the ecliptick, which is a great circle in the heavens, interſecting the equinoctial in two oppoſite points, at an angle of 23 deg. 29 min. Theſe points aſtronomers call *Aries* and *Libra*; and as the ſun is always in the ecliptick, and performs a revolution in it once a year; he will be twice in the equinoctial every year. Let us then ſuppoſe him in *Aries* the 21ſt of *March*, and at the ſame time on our meridian.

Now before the heavens has made one entire revolution, the ſun has moved near a whole degree eaſtward in the ecliptick, ſo that when *Aries* comes next day to the meridian, the heavens muſt turn near a whole degree more to the weſtward before the ſun comes to our meridian: Hence the point of the equinoctial that comes to the meridian with the ſun,

ſun, will be near a whole degree further from *Aries*, than that point of the equinoctial which was on the meridian with the ſun the preceding day, and ſo the ſun will be continually moving further from *Aries*, till, having performed his annual revolution, he again returns to the meridian with *Aries*.

It will be eaſy now to conceive what is meant by the ſun or ſtar's right aſcenſion; for it is the degrees and minutes of the equinoctial intercepted betwixt the firſt point of *Aries*, (or rather the point where the ecliptick interſects the equinoctial) and a meridian paſſing through the ſun or ſtar.

The right aſcenſion of the ſun and ſtars in the tables is given in time, allowing 15 deg. to one hour, which makes one degree of the equinoctial four minutes of time, and as the ſun's right aſcenſion, is given, when it is on the meridian at noon, it will be eaſy to find the hour when any ſtar comes on the meridian: For ſuppoſing the ſtar's aſcenſion 15 degrees, or one hour, and the ſun's aſcenſion two hours, it is plain the ſtar will come to the meridian one hour before the ſun that is at 11 hours after midnight, but if the ſtar's aſcenſion be 14 hours, and the ſun's 13, the ſtar will come to the meridian one hour after the ſun, that is at one hour after noon; hence

To find the Time of any Star's coming to the Meridian.

Rule. Subtract the ſun's right aſcenſion from that of the ſtar, or if the ſtar's right aſcenſion be leſs than the ſun's add 24 thereto, and the remainder is the time of the ſtar's coming to the meridian from noon; if the remainder exceeds 12, ſubtract 12 from it, the laſt remainder is the time from midnight.

Exam-

Example 1. What time will *Fomelhault* come to the meridian the 21ſt of *October*.

Examp, 2. What time will the *Bull's Eye* come to the meridian the 26th of *October*.

	H.	M.
From ſtars aſcen.	22	40
Sub. Suns.	13	44
Hour from noon	8	56

	H.	M.
To ſtars aſcen.	4	21
Add	24	00
	28	21
Suns aſcen.	14	03
	14	18
Subtract	12	
Hour from mid.	2	18

Having thus found at what hour the ſtar comes to the meridian, its altitude may be taken, and the latitude found as before directed, when the obſervation is by the ſun.

As it frequently happens that the heavens are ſome times obſcured, it will be neceſſary to know what ſtars come upon the meridian when an obſervation may be had, for which take the following rule.

To find what Star will come upon the Meridian at any propoſed Hour of the Night.

Rule. To the ſun's right aſcenſion add the time from noon at which the ſtars coming to the meridian is required, their ſum is the right aſcenſion of the ſtar that comes upon the meridian about that time, and this being found in the table, the ſtar correſponding thereto will be that required, and proper to obſerve by.

Ex-

Example. What ſtar will come upon the meridian the 7th of *April* about 8 at night?

	H.	M.
Sun's aſcen.	1	4
Time from noon	8	0
Stars aſcen.	9	4

The neareſt in the table is 9 h. 14 m. the aſcenſion of *Hydras* heart, which therefore comes upon the meridian at 8 h. 10 min.

SECT. III.

Of the Variation of the Compaſs.

The compaſs is the only inſtrument the mariner has to direct his courſe; but if the magnetick needle does not point out the true meridian, which is frequently the caſe, he will be led into a very great error if the variation is not known.

Now if the ſun or ſtar be on the meridian when the altitude is not above 15 degrees, the bearings may be taken by an azimuth compaſs, and the variation thereby diſcovered, this can happen by the ſun only to thoſe who live within the polar circles; we ſhall therefore ſhew how it may be found by the ſun's amplitude and azimuth.

When the ſun is in the equinoctial, it riſes in the Eaſt, and ſets in the Weſt, and then it has no amplitude; when it has North declination, it riſes and ſets to the Northward, but when South declination, it riſes and ſets to the Southward of the Eaſt and Weſt, and the degrees of the horizon intercepted betwixt the Eaſt, and the ſun when riſing, or the Weſt when ſetting, is the ſun's amplitude.

PROBLEM I.

The latitude and ſun's declination given; to find his amplitude.

Exam-

Example. Latitude 48 deg. 38 min. declination 22 deg. 18 min; required the ſun's amplitude? Or, which is the ſame thing, courſe 48 deg. 38 min. difference of latitude 3795; required the diſtance?

As co-ſine lat. 48° 38′	9.82012
Is to radius	10.
So is ſine declin. 22° 18′	9.57916
To ſine ampl. 35° 3′	9.75904
As co-ſine courſe 48° 38′	9.82012
Is to radius	10.
So is diff. of lat. 3795	3.57921
To diſt. 5743	3.75909

Demonſtration. R T, the parallel of declination, if North, cuts H O, the horizon in X, ſo ſhall C X be the amplitude equal to C Y, when the declination is South. *Plate* V. *Fig.* 1.

In the right angled triangle, C *f* X, equal C *o* Y; the angle at C is 48 deg. 38 min. that being the latitude, of which the angle at X is the complement. Now, if we call the angle *f* C X the courſe, and C *f* the difference of latitude, C X will be the diſtance, but C *f* is the natural right ſine of the declination, 3795; and, as we obſerved before, we ſhall have no occaſion to uſe the natural ſine, for it is plain, by the preceding operation, we take its logarithm, which may be had in the artificials, ſo the proportion will be, as above, co-ſine latitude, : R : : ſine declin : ſine ampl.

In this example, the true amplitude is 35 deg. 3 min. but if by obſervation it be found 16 deg. 3 min. Northerly or Southerly, according as the true is, their difference 19 deg. is the variation; but if either the true or magnetick be N. and the other S. as ſuppoſing the true 6 deg. 14 min. South, and the magnetick 5 deg. 1 min. North, the variation

is 11 deg. 15 min; after the variation is found, the courſe may be corrected; for the true courſe will be to the left of that ſteered when the variation is Weſterly, but to the right when Eaſterly, and to know whether the variation is Eaſterly or Weſterly, take this general rule.

When you are looking to the ſun at the time of obſervation, if the magnetick be to the right of the true, the variation is Weſterly; but if to the left, Eaſterly; thus, in the preceding example, if the amplitude was taken at ſun riſing, the true would be E. 35 deg. 3 min. N. and the magnetick being E. 16 deg. 3 min. the variation is 19 deg. Weſterly, and a ſhip then ſteering North, would make the true courſe N. 19 deg. W. If the obſervation was at ſun ſetting, the true is W. 35 deg 3 min. N. and magnetick W. 16 deg. 3 min. the variation is Eaſterly, and then the true courſe would be N. 19 deg. E; hence, if the variation is Weſterly, it muſt be allowed to the left, and if Eaſterly, to the right of the courſe ſteered.

PROBLEM II.

The latitude of the place, the ſun's declination and altitude given, to find his azimuth. This will admit of three different caſes, which may be ſolved by one general proportion; the third term of which muſt be obtained by a previous preparation.

As the comp. of the lat. and of the altitude are always given, we have the ſum or difference of theſe two arches, by addition or ſubtraction, and of conſequence the natural ſine of their ſum, or of their difference, and alſo the nat. ſine of their declination may be had in the table. (See Chap. III. Sect. 2.) and with theſe we are to operate as the circumſtances require.

CASE

CASE 1. Latitude and declination both North or both South, and the obſervation taken in the fore-noon after the ſun has paſt the prime vertical, that is, the azimuth circle paſſing through the Eaſt point of the horizon, or in the afternoon before the ſun comes to the Weſt; or, which is the ſame thing, when the ſun is betwixt the mid day meridian and prime vertical.

Here we muſt work by the nat. ſine of the difference of the two arches, *viz.* the altitude and complement of the latitude; which is to be added to the natural ſine of the declination, when the comp. of the latitude exceeds the altitude: but if the altitude be greateſt, the nat. ſine of their difference is to be ſubtracted from that of the declination, and the ſum or remainder will be the third term of the proportion.

CASE 2. Latitude and declination both North, or both South, and the ſun betwixt the prime vertical and midnight meridian.

Add the altitude to the complement of the latitude, and find the nat. ſine of the ſum of theſe two arches, from which ſubtract the nat. ſine of the declination, and the remainder will be the third term of the proportion.

CASE 3. Latitude and declination, one North and the other South. In this caſe the ſun is always betwixt the mid-day meridian and prime vertical, and the comp. of the latitude always exceeds the altitude, and when the difference of theſe arches is equal to the declination, the ſun is on the meridian. Here then ſubtract the natural ſine of the declination of the arches, from the nat. ſine of the difference of the arches, and the remainder will be the third term of the proportion. *Note*, If the comp. of the latitude be equal to the altitude, the artificial ſine of the declination will be the third term of the proportion:

tion: the general proportion is as co-ſine of the altitude is to the ſecant of the latitude; ſo is the third term, found as before directed, to the verſed ſine of the azimuth.

It is preſumed, that the variation may be obtained to leſs than $\frac{1}{4}$ of a point, by Mr *Mountaine*'s variation chart; ſo the obſerver may know whether the ſun is betwixt the mid-day or mid-night meridian and the prime vertical, even by the ſteering-compaſs; however, if the reſult of the operation ſhould exceed the radius, from the natural number of that logarithm ſubtract the radius, which is always 1000; the logarithm of the remainder will be the artificial co-ſine of the azimuth, which muſt be taken from the midnight meridian, if we uſe the nat. ſine of the difference of the arches; but if we work by the nat. ſine of the ſum of the arches, the azimuth muſt be taken from the mid-day meridian. It muſt be obſerved, if the third term conſiſts of four figures, 9 muſt be the index of the logarithm; if three figures, 8 muſt be the index; if two figures, 7; and when it is only one figure, 6 muſt be the index of the logarithm. Hence we may deduce the following general rule: To the logarithm of the third term, with the proper index prefixed, add the artificial ſecant of the latitude, and from the ſum ſubtract the artificial co-ſine of the altitude, the remainder will be the logarithm of the verſed ſine of the azimuth. If we have not a table of verſed ſines, we may ſubtract 10000 from the natural number of its logarithm, and the logarithm of the remainder will be the artificial co-ſine of the azimuth. We ſhall illuſtrate the whole by an example in each caſe.

Example 1. Latitude 48 deg. 38 min. North; declination 22 deg. 18 min. North; altitude 41 deg. 22 min. equal to the comp. of the latitude; therefore co-ſine

co-ſine altitude : ſecant latitude : : ſine declination : verſine azimuth.

To third term ſine declination	22°	18′	9.57916
add ſecant latitude	48	38	10.17988
Sum — —			19.75904
Co-ſine altitude	41°	22′	9.87535
	from ſouth.		
7651 N. verſine of azimuth	76	24	9.88369
2349 N. co-ſine of azimuth	76	24	9.37088

Example 2. Altitude 15 deg. latitude and declination as before.

Preparation.

To complem. lat.	41°	22′		
add altitude	15	00		
Sum	56	22 N.	ſine	8326
Declination	22	18 N.	ſine	3794
Difference of the ſines			4532	9.65629
Secant latitude	48°	38′		10.17988
			Sum	19.83617
Co-ſine altitude	15 deg.			9.98494
			°	′
7100 N. ver-ſine of azim. from N.			73	8
2900 N. co-ſine of azim. from N.			73	8
				9.85123
				9.46240

Becauſe it may be doubtful on which ſide of the prime vertical the ſun may be, at the time of obſervation, we ſhall work this example by the nat. ſine of the difference of the arches.

Complement

Comp. lat.	41°	22′			
Altitude	15	00			
Difference	26	22	N.S.	4441	
Declination	22	18	N.S.	3794	
Sum of N. fines				8235	9.91566
Secant latitude	48°	38′			10.17988
					20.09554
Co-fine altitude					9.98494
12900 is N. number					10.11060
10000					
2900 N. co-fine as before					9.46240

Example 3. Latitude 48 deg. 38 min. North; Declination 22 deg. 18 min. South; altitude 15 deg. Required the fun's azimuth?

In this, and in all fuch cafes, the fun is betwixt the prime vertical and the mid-day meridian, all the time it is above the horizon; and as the altitude can never exceed the complement of the latitude, we may ufe the difference of the fines.

Comp. latitude	41°	22′			
Altitude	15	00			
Difference	26	22	N.S.	4441	
Declination	22	18	N.S.	3794	
				647	8.81090
Secant latitude	48	38			10.17988
					18.99078
Co-fine altitude	15 deg.				9.98494
1014 N. ver-fine of	26 deg.	2 min.			9.00584
8986 Comp.	26	2			9.95356

After finding the true azimuth, the magnetick may be had by obſervation, and the courſe corrected as before by the amplitude.

Having now ſhewn how to find the latitude and variation of the compaſs by cœleſtial obſervation, our next buſineſs ſhall be to conſider how far ſuch obſervations may aſſiſt us in determining the longitude at ſea: In order to which, it is neceſſary to know the exact time even to minutes and ſeconds of an hour, both at the place the ſhip is in, and likewiſe of ſome other place, whoſe longitude is known; and that at the very inſtant of time the obſervation is taken: We ſhall therefore ſhew how to find the hour at ſea.

PROBLEM III.

Latitude, altitude and declination given to find the hour; in this we ſhall make two different caſes, which may be ſolved by one proportion; but here, as in the azimuths, the third term muſt be found by a previous preparation, which we ſhall illuſtrate by the ſame examples taken before for the azimuths.

CASE 1. When the obſervation is taken before ſix in the morning or after ſix at night, ſubtract the declination from the comp. of the latitude, add to the nat. ſine of the remaining arch, and the nat. ſine of the altitude, and their ſum will be the third term; this can only happen when the latitude and declination are both North or both South, as in the 2d example.

CASE 2. When the obſervation is taken after ſix in the morning, or before ſix at night, as in the 1ſt and 2d examples, ſubtract the nat. ſine of the preſent altitude, from the nat. ſine of the meridian altitude for that day, and the remainder will be the third term; if it be doubtful whether it is paſt ſix or not at the time of obſervation, we may uſe

uſe either of theſe; and if the reſult ſhould exceed the radius, ſubtract the radius from it, as before for the azimuth, and the logarithm of the remainder will be the artificial ſine of the hour before ſix, if we work by the difference of the nat. ſines of the preſent and meridian altitude; but if we work by the ſum of the ſines as in the preceding caſe, it will be paſt ſix. After the third term is found, uſe the following proportion; co-ſine declin : ſect. lat : : third term : ver-ſine of the hour. Hence the following general rule.

To the logarithm of third term, add the artificial ſect. of the latitude, and from the ſum ſubtract the artificial co-ſine of the declin. and the remainder will be the logarithm of the verſed ſine of the hour. *Plate* V. *Fig.* 1.

EXAMPLE 1.

H Æ comp. lat.	41°	22′ N		
Æ R declin.	22	18 N		
H R mer. alt.	63	40 N. S.	8962	
H Æ pre. alt.	41	22 N. S.	6609	
Diff. ſines is the third term			2353	9.37162
Secant lat. 48 deg. 38 min.				10.17988
Sum				19.55150
Co-ſine declin. 22 deg. 18 min.				9.96624
N. ver-ſine of 52° 02′ is			3848	9.58526
Comp. is	37	58 N. S.	6152	9.78902
52 deg. 2 min. is before 12			3 h. 28 m. 8 ſ.	
37 deg. 58 min. is after ſix			2 h. 31 m. 52 ſ.	
So the hour at the time of obſervation is			8 h. 31 m. 52 ſ.	

EXAMPLE

EXAMPLE 2.

Comp. lat.	41°	22′ N		
Declin.	22	18 N		
Rem.	19	04	N. S. 3267	
Pre. alt.	15	00	N. S. 2588	
Sum of the ſines			5855	9.76752
Sect. lat. 48 deg. 38 min.				10.17988
Sum				19.94740
Co-ſine declin. 22deg. 18 min				9.96624
N. ver-ſine of 87 deg. 34 min. is			9576	9.98116
Comp. is	2. 26 N. S.		424	8.62737
Here 2 deg, 26 min. before ſix				0 h. 9 m. 44 ſ.
87 h. 34 m. paſt midnight				5 h. 50 m. 16 ſ.
				6 h. 0 min. 0 ſ.

EXAMPLE 3.

Here the meridian altitude is the ſame with the remainder in the preceding.

Merid. alt.	19°	4′ N. S.	3267	
Pre. alt.	15	00 N.	2588	
Diff. of ſines			679	
Co-ſine declin.	22°	18′		9.96624
Sect. lat.	48	31		10.17988
Diff. Sines		679		8.83187
				19.01175
N. ver-ſine of	27°	15′ is	1110	9.04551
Comp. is	62	45 N. S.	8890	9.94890

Demonſtration. Draw the parallels of altitude Æ K, and A B, alſo the parallels of declination R T S and

and U V, so shall D be the sun's place in the heaven, at the first observation; Æ D the versed sine of the azimuth to the radius Æ G, and R D the versed sine of the hour, to the radius R *f*, and drawing an azimuth circle through D, it will intersect the horizon in F, so shall C F be the co-sine of the azimuth, which may be measured on the line of sines, but this may be done without drawing the azimuth circle. Thus, 1st, Draw a radius to meet the parallel of altitude in the circumference, which, in the 1st Example, will be C Æ, because the complement of the latitude and of the altitude are equal. 2dly, Draw D E parallel to C Z, to intersect C Æ in *h*; so shall C *h* be equal to C F the co-sine, and Æ *h* equal to H F, the versed sine of the azimuth, for as the radius of any parallel of altitude, is to the radius of the horizon, so is the sine of any arch in the prrallel of altitude, to the sine of a similar arch in the horizon; (*see Chap.* III. *Sect.* II.) now the triangles Æ C G, and Æ *h* D being similar Æ G : (the radius of the parallel) Æ C : : (equal C H the radius of the horizon) Æ D : Æ *h* the versed sine of the azimuth; again in the triangle Æ *k* D, the angle *k* Æ D is equal to the angle P C O, the given latitude, for Æ *k* is parallel to P C, and Æ K to H O; the side Æ D is the versed sine of the azimuth to the radius Æ G; the side Æ *k* is the right sine of the declination, therefore making Æ *k* radius, Æ D will be the secant of the latitude. Now to find Æ D it will be R : sect. lat : : Æ *k* : Æ D, the operation is

Radius			10:
Sect.	48	38	10.17988
Sine	22	18	9.57916
Æ D			9.75904

Pl. V. *Fig.* 1.

Co-

Co-fine alt. 41 d. 22 m.	9.87535
Radius	10.
Æ D	19.75904
Æ *b* verſed fine azimuth	9.88369

Here are two proportions, but as in the firſt the logarithm of the radius is ſubtracted from the ſum of the logarithms, of the ſect. of the lat. and of the ſine of the declin, the remainder gives the logarithm of Æ D, now in the ſecond, the logarithm of the radius is again added to that of Æ D; ſo the radius may be taken away in both, and then the operation may be done by one proportion as before.

Again, if we draw a meridian through D, it will cut the equinoctial in *q*; ſo ſhall C *q* be the ſine of the hour from ſix, and Æ *q* the verſed ſine, which may be found without drawing the meridian after the ſame manner the azimuth was found; thus, 1ſt, Draw a radius C R to meet the parallel of declination in the circumferenee; 2dly, Draw D *p*, parallel to P S, to interſect C R in *p*; ſo ſhall C *p* be equal to C *q* and R *p* equal to Æ *q* the verſed of the hour, (*ſee the demonſtration for the azimuth.*) Now R *e* is the ſine of the meridian altitude, *a e* the ſine of the preſent altitude, and R *a* their difference. So, R : Sect. lat : : R *a* : R D = R *a* × Sect lat ÷ R and co-ſine declin : R : : R D : verſed ſine hour, and omitting the radius in both, we have, as co-ſine declination, is to ſecant latitude, ſo is R *a*, to the verſed ſine of the hour, as before.

In the 2d Example, *g* is the ſun's place in the heavens at the time of obſervation; B *g* is the verſed ſine of the azimuth to the radius *n* B and T *g* the verſed ſine of the hour to the radius *f* T; Q O is the complement latitude, O B the altitude, the arch Q O B is their ſum, of which B M is the N. ſine;

ſine; *y* M the N. ſine of the declination and B *y* their difference. So, in the triangle B *g y*, R: Sect lat:: B *y*: B *g*; and as co-ſine altitude is to radius, ſo is B *g* to the verſed ſine of the azimuth, and omitting the radius in both, we have the general proportion, as before.

Again, Q O is the complement latitude, Q T the declination, and T O their difference, to which adding O B the altitude, we have the arch T O B, of which T B is the right ſine. In the triangle T B *g*, R: Sect lat:: T B: T *g*, the verſed ſine of the hour, to the radius T *f*; and co-ſine declination: R:: T *g*: verſed ſine hour, and omitting the radius we have the general proportion.

In the 3d Example, H Æ is the complement latitude, H A the altitude, and the arch A Æ their difference, of which A *t* is the right ſine, from which ſubtracting the ſine of the declination we have A *x*, ſo A *m* may be found as before; in like manner in the triangle U 1 *m*, U 1, is the difference betwixt the ſine of the meridian, and the ſine of the preſent altitude, and U *m* the verſed ſine of the hour.

Tho' we may find the azimuth and hour by the preceding problems, yet as the common books of navigation have neither the natural or verſed ſines, it will be neceſſary to ſhew how they may be obtained by the common tables; and as the rules are founded on the ſtereographick projection of the ſphere, we ſhall refer for the demonſtrations to the authors who have treated of ſphericks: but here it will be proper to remark, that the quadrant gives only the apparent altitude; we muſt therefore find the true by the following method.

1. Add 16 min. for the ſun's ſemidiater, to that on the quadrant, if you obſerve by the lower edge; but if by the upper, ſubſtract 16 min.

2. Sub-

2. Subtract the refraction, and the allowance for the height of the eye above the horizon from the above ſum or remainder. *Note*, if we uſe a back obſervation, the allowance for the height of the eye muſt be added.

Another thing to be obſerved is, that the declination in the tables is calculated for mid-day, and, therefore, a proportional part of the difference betwixt that, and the declination the preceding day, muſt be added or ſubtracted to that in the tables. The latitude muſt likewiſe be had at the time of obſervation, by working for the latitude made by account ſince laſt obſervation. We ſhall illuſtrate this by the following example:

June 3, 1759. Latitude obſerved 49 deg. 2 min. apparent altitude 14 deg. 53 min. Required the true?

appar. altitude of ſun's	lower edge	14°	53′
	add ſemidiameter	0	16
	center	15	09
Height of eye for 25 feet	0 6′	Subtract	9
Refraction —	0 3		
True altitude of the ſun's center		15°	0

The declination the ſecond day is 22 deg. 12 m. and the third 22 deg. 20 min. the difference is 8; one quarter of which, 2, being ſubtracted from 20, becauſe it wants only ſix hours of noon, makes the declination at the time of obſervation 22 deg. 18 min. Again, the ſhip is ſuppoſed to have made 24 miles ſouthing ſince laſt obſervation, which makes the preſent latitude 48 deg. 38 min.

Having thus found the true altitude, latitude, and declination, the azimuth and hour may be found by the following rule:

1. Add the comp. of the latitude, comp. of the altitude, and comp. of the declination into one ſum.

2. From

2. From half that sum subtract the complement of the declination, if you work for the azimuth; but if for the hour, subtract the complement of the altitude from the half sum; and note the half sum, and remainder.

3. Take the artificial sines of the half sum, and of the remainder.

4. Add to these two artificial sines, the complement arithmetical of the artificial sines, of the comp. of the altitude and comp. of the latitude, or which is the same thing, the secants of the altitude and of the latitude, omitting the index; half the sum of these four logarithms is the co-sine of half the sun's azimuth from the North, in North latitude, and from the South, in South latitude. But, to find the hour, instead of the secant of the altitude, take that of the declination, then work as for the azimuth, so shall, half the sum of the four logarithms be the co-sine of half the hour from noon.

Note, If either the altitude or declination be one North, and the other South, instead of taking the complement of the declination, there must be 90 degrees added thereto, and then proceed as before.

Operation for the Azimuth.

Comp. alt.	75° 00′	Sec. 15° 00′	0.01506
Comp. lat.	41 22	Sec. 48 38	0.17988
Comp. dec.	67 42		
Sum	184 04		
Half sum	92 02,	Sine 87 58	9.99973
Comp. dec.	67 42		
Remainder	24 20	Sine —	9.61494
Sum of the four logarithms			19.80961
Half sum co-sine		36° 34′	9.90481
		36 34	
Azimuth from the North		73 08	

Operation

Operation for the Hour.

Comp. dec.	67°	42′	Sect. 22°	18′	0.03376
Comp. lat.	41	42	Sect. 48	38	0.17988
Comp. alt.	75	00			
Sum	184	04			
Half sum	92	02	fine 87°	58′	9.99973
Comp. alt.	75	00			
Remainder	17	02	Sine	—	9.46676
Sum of the four logarithms					19.68013
Co-fine	46°	13′	Half sum		9.84006

46 13

92 26, at four deg. to one min. of time.

4

h.	m.	s.	
6	9	44	if in the afternoon,
5	50	16	if in the morning.

Becauſe the half ſum exceeds 90 deg. we take the ſine of the ſupplement, which is the ſame with that of the half ſum by Definition 4, Chap. 11, Sect. IV.

Comparing theſe with the operations, by the nat. ſines, we ſhall find the anſwers exactly the ſame; ſo that it is indifferent whether we work by the one or the other: this laſt is the method generally practiſed, and tho' it may be allowed the moſt expeditious, yet as the demonſtrations would require a larger treatiſe than is conſiſtent with the conciſeneſs of our Rudiments, we hope the candid reader will excuſe our treating ſo fully of the natural ſines, whereby

whereby the whole of navigation may be performed by right angled triangles alone, as was at firſt propoſed; and in calculations for the hour it would be proper to work both ways, becauſe this is of great importance in determining the longitude,

There are aſtronomical tables, calculated to great accuracy, which give the exact time when the eclipſes of Jupiter's ſatellites ſhall happen at all the royal obſervatories in Europe. Now, as theſe eclipſes are ſeen, in different places, at the ſame inſtant of time, we need only find the exact time in thoſe places, at the inſtant the tranſits are ſeen, by the preceding problem; for then the difference of time, in hours, minutes, and ſeconds, being multiplied by 15, would be the difference of longitude, in degrees, &c. betwixt the place of obſervation, and the obſervatory, from which the longitude is accounted; and it is, by this means, the longitude of any place is determined on land; but as the ſatellites cannot be ſeen by the naked eye, the inſtant of the tranſits at ſea cannot be obtained, to that nicety, requiſite for determining the longitude, unleſs ſome expedient be found to prevent the ſhip's motion from affecting the obſerver.

How far Mr *Irwin*'s Marine Chair may anſwer this end, our readers may judge by what he has publiſhed on that ſubject. It is certain he made an experiment of it on board his Majeſty's ſhip *Magnanime*, which give great hopes of ſeeing that important point determined.

SECT. IV.

Deſcription and Uſe of Davis's *and* Hadley's *Quadrants.*

Davis's quadrant conſiſts of two arches, framed and faſtened as in *Fig.* 2. *Plate* VI. Both the arches are

are drawn from the ſame center; that of the ſhorteſt radius contains 60, and that of the longeſt 30 deg. In order to graduate the arches, from the center H, and with the radius H G, taken from any line of chords, deſcribe a quarter of a circle; ſo the angle M H G will be 90 degrees, which may be tranſferred from the dotted arch to thoſe on the inſtrument; the diviſions of the 30 arch beginning in the line H G, and the diviſions of the 60 arch in the line H M; ſo ſhall the two arches make 90 deg.

It has three vanes, *viz.* the horizon vane, fixed at H; the ſight vane, to move on the 30 arch; and the ſhade vane, to move on the 60 arch; as the circumſtances require. The inſtrument ſhould be in ſuch a poſition, at the time of obſervation, that the line K H paſſing from the eye to the ſlit in the horizon vane, may be parallel to the horizon, when the ſhadow of the ſun by the ſhade vane coincides with the upper part of the ſlit; for then counting the degrees on both arches, their ſum will be the ſun's zenith diſtance.

Demonſtration. Let the line W X be parallel to the horizon; the angles Z H M and M H S, both together, will be the ſun's zenith diſtance; but M H Z is equal to K H G; for if either of theſe two angles be added to the angle M H K, their ſum will be 90 degrees; therefore the ſum of the degrees on both the arches will be the ſun's zenith diſtance.

Hadley's quadrant is the beſt inſtrument for taking altitudes at ſea, for the deſcription of which we refer to a pamphlet, publiſhed by Mr *Watkins*, optician, at *Charing-Croſs*, which explains the uſe and theory of that noble inſtrument in a plain familiar manner, intelligible to any ordinary capacity: but as the degrees on the arch are only divided into three equal parts, which are 20 minutes each, it will

be neceſſery to ſhew how the altitude may be taken to minutes, and what to allow for the height of the eye above the viſible horizon. *Pl.* VI. *Fig.* 3.

Let A B be a part of the arch of *Hadley*'s quadrant divided into whole degrees, and theſe ſubdivided into three equal parts, which will be 20 min. each. Let C D, the chamfered edge of the index, contain 3 deg. which will therefore be 9 diviſions of the arch on the quadrant, that is, 180 min. Now if this ſpace on the index be divided into ten equal parts, they will be 18 min. each; and whereas thoſe on the arch are 20 minutes, it is plain, when the line in the middle of the index unites exactly with any one of the diviſions on the arch, ſo as to form a ſtrait line, it then points out either whole degrees, or ſubdiviſions, which will be either 20 or 40 minutes more, to be added to the degrees; in that poſition the other diviſions of the index will fall ſhort of thoſe upon the arch; that marked 2 will be two minutes ſhort of the firſt diviſion on the arch, next to that which coincides with the line in the middle of the index: the diviſion marked 4 on the index, four minutes ſhort of the ſecond on the arch, &c. ſo that if the index be moved till 2, 4, 6, 8, or 10 coincide with any of the diviſions of the arch, then 2, 4, 6, 8, or 10 min. muſt be added to thoſe that would have been expreſſed by the index, had, the line in the middle of it, coincided with any of the diviſions of the arch; and for the ſame reaſon, when 12, 14, &c. coincide with any of the diviſions of the arch, there muſt be 12, 14, &c. minutes added. Hence, as it ſtands in the figure, the index expreſs 2 degrees and 50 minutes.

Now, if inſtead of 9 diviſions of the arch of the quadrant, we take 19, and divide that ſpace on the index into 20 equal parts, they will be 19 minutes each; by which means the altitudes may be taken

to

to one minute; and as to the ſeconds (on the land quadrants) there is a braſs nut fitted to ſlide along the arch, which may be faſtened where moſt convenient; there is alſo a ſmall plate, like the dial-plate of a watch, fixed to the nut; through the center of this plate goes a ſcrew, with a ſmall index, which moves round the plate, while the index of the quadrant is moved one minute by turning the ſcrew; and the dial-plate being divided into 60 equal parts, its index will point the ſeconds, and the diviſions, on the index of the quadrant, the minutes. *Plate* VI. *Fig.* 3.

The viſible and rational horizon, being always parallel to one another, and the eye of the obſerver above the viſible, as the quadrant ſtands in the figure, the ſight-vane muſt be moved up to A, before the horizon can be ſeen, ſo the zenith diſtance by the quadrant will exceed the true by the angle A H K.

In order to find this angle, let *m n* be a part of the circumference of the terraqueous globe; R the center; this, in the plate, is in the line Y Z. Now, ſuppoſing the eye at H, 40 feet above the ſurface of the ſea, the horizon will then be ſeen at *m*; H T will be a tangent to the ſurface of the globe in the point T, and the angle M H T equal to the angle H R T. In the right angled triangle H R T is given R T, the ſemi-diameter of the earth, which ſuppoſe 21123276 feet, and R H will be 40 feet more, that is, 21123316; therefore making T R radius, H R will become the ſecant of the angle at R, and the proportion will be R T : R H :: R : ſecant of the angle.

OPERATION.

R T : 21123276	7·324728
R H 21123316	17·324729
Secant of 0° 7′	10.000001

Now

16-004

Now becauſe the angle at T is 90 degrees, the angles T H R and T R H both together make 90 degrees; but the angles T H R and T H M both together make 90 deg. therefore the angle T R H is equal to the angle (T H M) A H K. By this operation, the ſun's zenith diſtance, as the quadrant gives it, exceeds the true 7 minutes, which muſt therefore be ſubtracted from it; or if we take its altitude, we muſt add 7 minutes to that on the quadrant.

It muſt be obſerved, this is when we uſe the back obſervation; but when we take the altitude by the fore obſervation, as ſuppoſing the eye at H, that on the quadrant will be the arch S *p*, whereas the true is S *q*; their difference is the arch *q* H *p*, 7 minutes, as by the foregoing operation; hence, in the fore obſervation, which is that generally uſed by *Hadley*'s quadrant, we muſt ſubtract the allowance for the height of the eye from that on the quadrant.

Note, The proportion that 40 feet has to the diameter of the earth, is ſo ſmall, that it cannot be repreſented in the plate, the deſign of the figure being only to demonſtrate the reaſon of the operation; and an error in any of the figures which we have taken for the ſemi-diameter of the earth, will not alter the angle half a minute; ſo the tables of the dip of the horizon may be allowed ſufficiently accurate for the mariner, as to what concerns the latitude.

F I N I S.

Plate I.

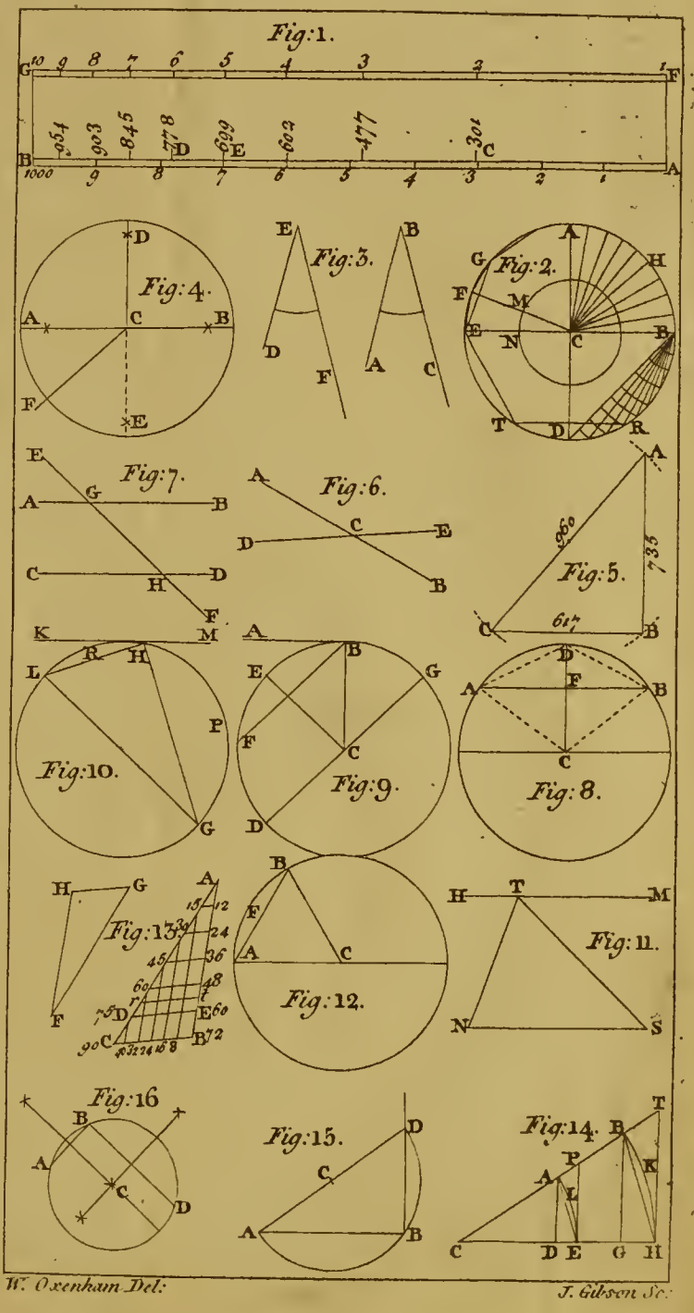
Fig:1.
Fig:2.
Fig:3.
Fig:4.
Fig:5.
Fig:6.
Fig:7.
Fig:8.
Fig:9.
Fig:10.
Fig:11.
Fig:12.
Fig:13.
Fig:14.
Fig:15.
Fig:16
W. Oxenham Del:
J. Gibson Sc:

RPJCB

Plate II.

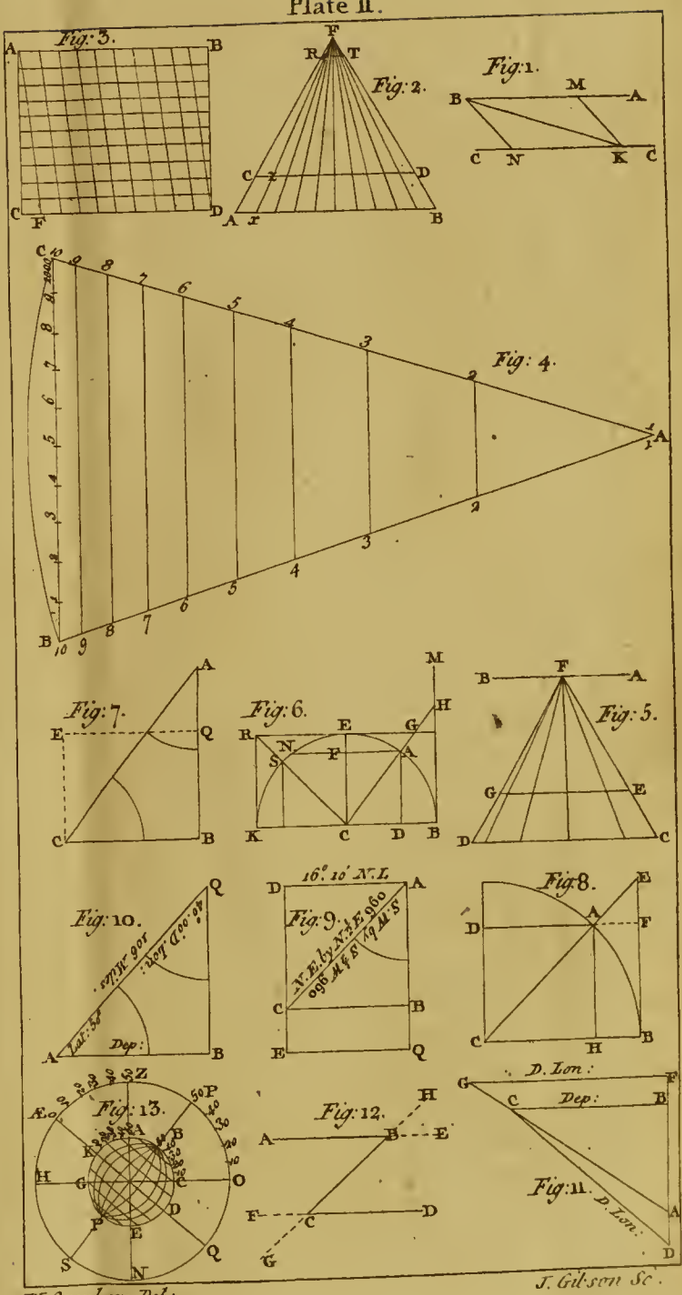
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Fig: 2.
Fig: 1.
Fig: 4.
Fig: 7.
Fig: 6.
Fig: 5.
Fig: 10.
Fig: 9.
Fig: 8.
Fig: 13.
Fig: 12.
Fig: 11.
D. Lon:
Dep:
W. Oxenham Del:
J. Gilson Sc.

R.P.J.C.B.

Plate III.

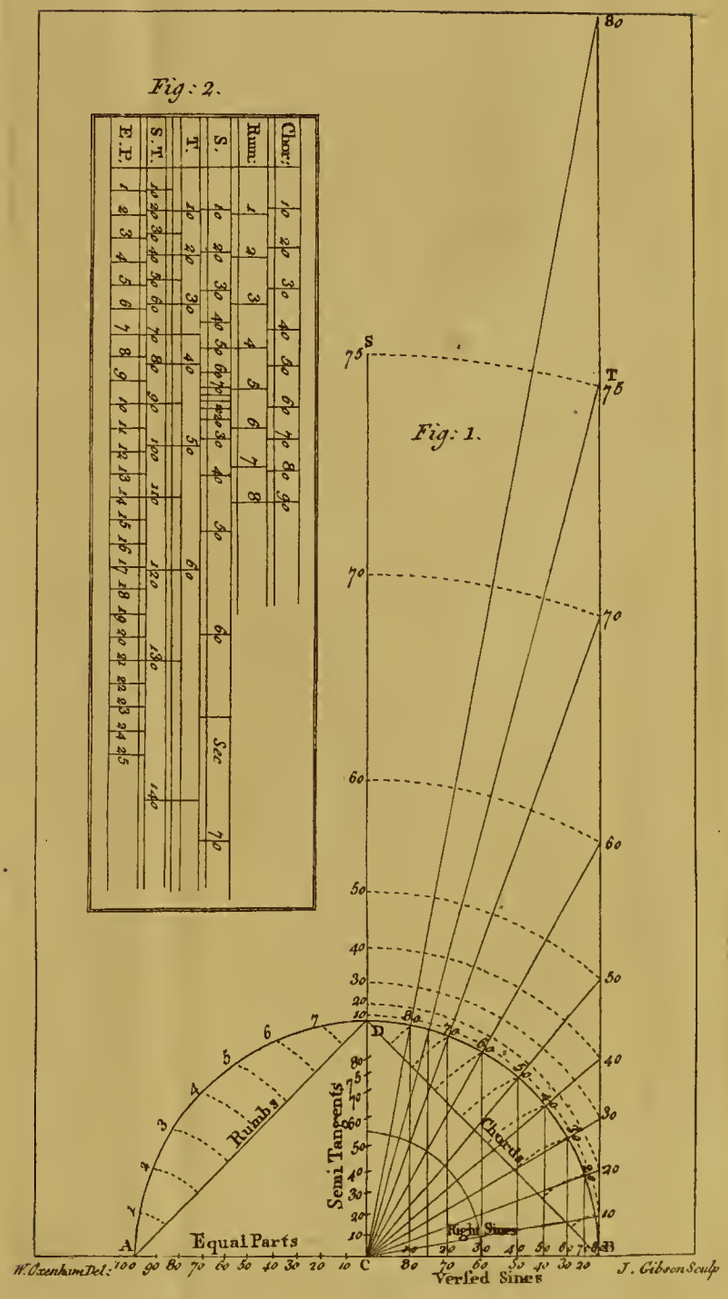
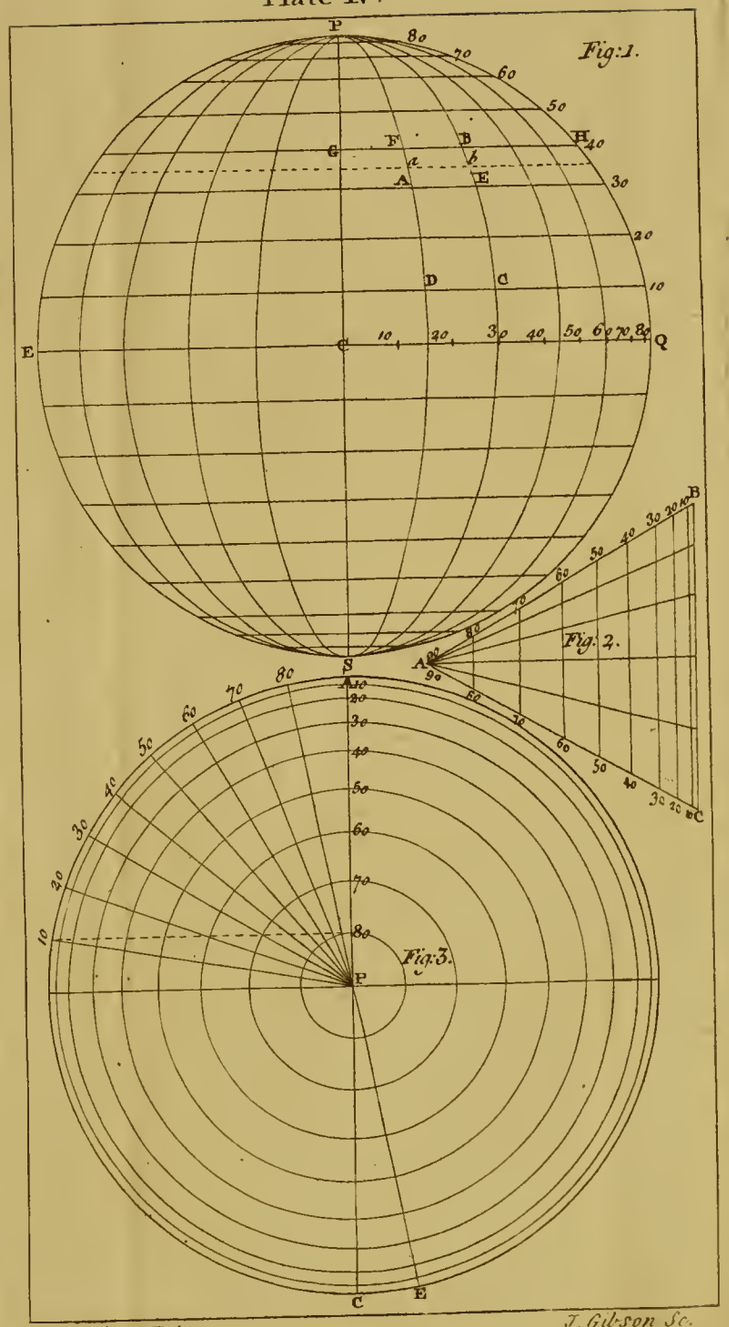


Fig: 2.
Chor:
Rum:
S.
T.
S.T.
E.P.
Fig: 1.
Rumbs
Equal Parts
Semi Tangents
Chords
Right Sines
Versed Sines
W. Oxenham Del:
J. Gibson Sculp

RPJC

Plate IV.

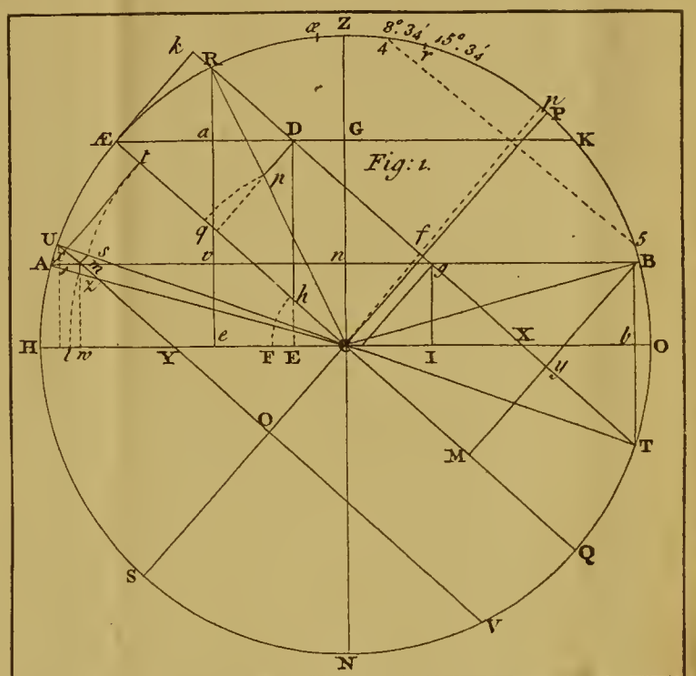


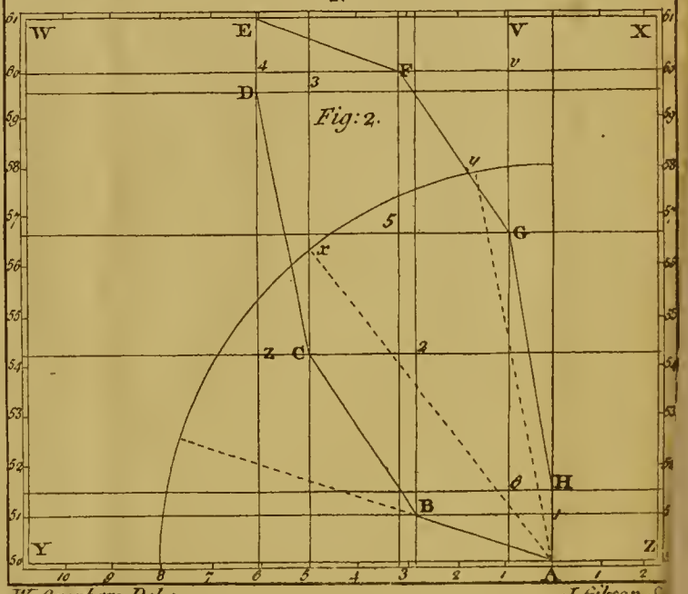
W. Oxenham Del:

T. Gibson Sc.

RPJCB

Plate V.





W. Oxenham Del.

J. Gibson Sc.

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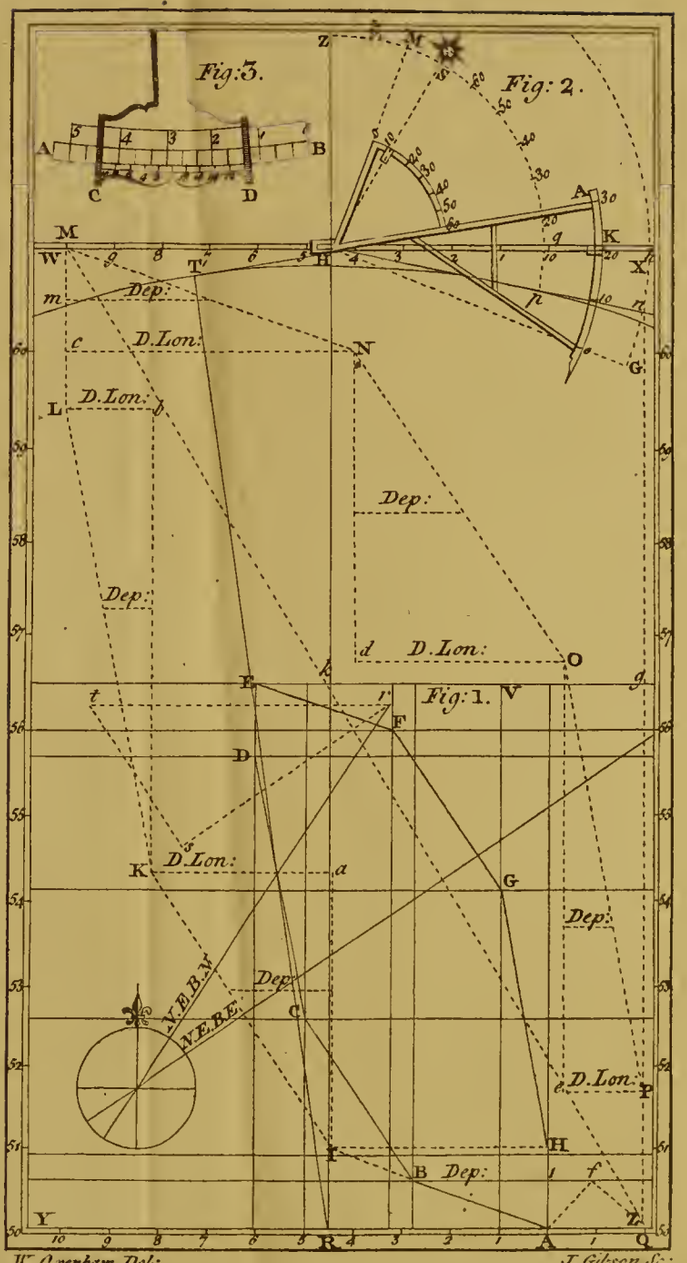
Plate VI.



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J. Gibson Sc:

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